LAZER DİYOT DENKLEMLERİNİN VOLTERRA SERİSİ İLE ÇÖZÜMLERİ

Prof. Dr. Remzi YILDIRIM

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VOLTERRA SERIES EXPANSION OF LASER DIODE RATE EQUATION

The basic laser diode equations are:

$$P' = \Gamma A(N - N_{tr})(1 - \hat{\varepsilon}P)P - \frac{1}{\tau_p}P + \frac{\beta\Gamma N}{\tau_n} \tag{1}$$

$$N' = \frac{1}{qV}I - \frac{1}{\tau_n}N - \Gamma A[N - N_{tr}][1 - \hat{\varepsilon}P]P \tag{2}$$

The expansion of equation (1) is:

$$P' = \Gamma A[N - N_{tr} - \hat{\varepsilon}PN + \hat{\varepsilon}N_{tr}P]P - \frac{1}{\tau_p}P + \frac{\beta\Gamma N}{\tau_n}$$

$$= \Gamma ANP - \Gamma AN_{tr}P - \Gamma A\hat{\varepsilon}NP^2 + \Gamma A\hat{\varepsilon}N_{tr}P^2 - \frac{1}{\tau_p}P + \frac{\beta\Gamma N}{\tau_n}$$

$$P' = \frac{\beta\Gamma}{\tau_n}N - (\Gamma AN_{tr} + \frac{1}{\tau_p})P + \Gamma A\hat{\varepsilon}N_{tr}P^2 + \Gamma ANP - \Gamma A\hat{\varepsilon}NP^2 \tag{3}$$

Adding equation (1) and (2)

$$P' + N' = \Gamma A(N - N_{tr})(1 - \hat{\varepsilon}P)P - \frac{1}{\tau_p}P + \frac{\beta\Gamma N}{\tau_n} + \frac{1}{qV}I - \frac{1}{\tau_n}N - \Gamma A(N - N_{tr})(1 - \hat{\varepsilon}P)P \tag{4}$$

$$P' + N' = \frac{I}{qV} - \frac{1}{\tau_n}(\beta\Gamma - 1)N - \frac{1}{\tau_p}P \tag{5}$$

The input current is composed of the sum of I_0 , D.C component , $\hat{I}$,a time –varying component. We thus let

$$I = I_0 + C\hat{I} \tag{6}$$

The Volterra expansion of the photon density and the electron density are:

$$P = \sum_{n=0}^{\infty} c^n P_n \quad \text{and} \quad N = \sum_{n=0}^{\infty} c^n N_n$$

substituting this expansion in equation (3) we obtain

$$\sum_{n=0}^{\infty} c^n P'_n = \frac{\beta\Gamma}{\tau_n}\sum_{n=0}^{\infty} c^n N_n - (\Gamma AN_{tr} + \frac{1}{\tau_p})\sum_{n=0}^{\infty} c^n P_n +$$

$$+\Gamma A\hat{\varepsilon}N_{tr}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}c^{n_1+n_2}P_{n_1}P_{n_2}$$

$$+\Gamma A\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}c^{n_1+n_2}N_{n_1}P_{n_2}$$

$$-\Gamma A\hat{\varepsilon}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}c^{n_1+n_2+n_3}N_{n_1}P_{n_2}P_{n_3} \qquad (7)$$

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$$\left[\sum_{n=0}^{\infty}C^{n}x_{n}\right]^{2}=\sum_{n_1=0}^{\infty}C^{n_1}x_{n_1}\sum_{n_2=0}^{\infty}C^{n_2}x_{n_2}=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}C^{n_1}x_{n_1}C^{n_2}x_{n_2}=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}C^{n_1+n_2}x_{n_1}x_{n_2} \qquad (8)$$

$$=a_0c^0+a_1c^1+a_2c^2+a_3c^3+a_4c^4+a_5c^5+..... \qquad (9)$$

$n_1+n_2=0\rightarrow c^0$
$n_1+n_2=1\rightarrow c^1$
$n_1+n_2=2\rightarrow c^2$
$n_1+n_2=3\rightarrow c^3$
$n_1+n_2=4\rightarrow c^4$
$a_0=x_0x_0=x^2$
$a_1=x_0x_1+x_1x_0=2x_0x_1$
$a_1=x_1x_1+x_0x_2+x_2x_0=x_1^2+2x_0x_2$

a_3	n_1	n_2	n_3
1	0	0	3
2	0	3	0
3	3	0	0
4	1	1	1
5	1	0	2
6	1	2	0
7	0	1	2
8	0	2	1
9	2	1	0
10	2	0	1
Degiskenler	N_{n1}	P_{n2}	P_{n3}

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$$m=\frac{(n+k-1)!}{(k-1)n!} \qquad (10)$$

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$$m=\frac{(n+k-1)!}{(k-1)n!}=\frac{3+3-1}{(3-1)3!}=10$$

$$a_3 = 2P_3 + 2P_0 + 2P_1 + 2P_2$$
$$N_0P_3 + P_0N_3 + N_1P_2 + N_2P_2$$
$$N_0P_0P_3 + N_0P_0P_3 + P_0P_0N_3 + N_1P_1P_1 +$$
$$N_1P_0P_2 + N_1P_0P_2 + N_0P_1P_3 + N_0P_1P_2 + P_0P_1P_2 + P_0P_1P_2$$

$$a_3 = 2(\;)P_3 + 2P_0 + 2P_1 + 2P_2 +$$
$$N_0P_3 + P_0N_3 + N_1P_2 + N_2P_2$$
$$N_0P_0P_3 + N_0P_0P_3 + P_0P_0N_3 + N_1P_1P_1 +$$
$$N_1P_0P_2 + N_1P_0P_2 + N_0P_1P_3 + N_0P_1P_2 + P_0P_1P_2 + P_0P_1P_2$$

we now equate like power of "c" to obtain

For c^0

$$P_0' = \frac{\beta\Gamma}{\tau_n}N_0 - (\Gamma A N_{tr} + \frac{1}{\tau_p})P_0 + \Gamma A\hat{\varepsilon}N_{tr}P_0^2 + \Gamma A N_0 P_0 - \Gamma A\hat{\varepsilon}N_0P_0^2 \qquad (4A)$$

For c^1

$$P_1' = \frac{\beta\Gamma}{\tau_n}N_1 - (\beta\Gamma N_{tr} + \frac{1}{\tau_p})P_1 + 2\Gamma A\hat{\varepsilon}N_{tr}P_0P_1 + \Gamma A\left[N_0P_1 + N_1P_0 - \Gamma A\hat{\varepsilon}\left[2N_0P_0P_1 + P_0^2N_1\right]\right] \qquad (4B)$$

For c^2

$$P_2' = \frac{\beta\Gamma}{\tau_n}N_2 - (\beta\Gamma N_{tr} + \frac{1}{\tau_p})P_2 + \Gamma A\hat{\varepsilon}N_{tr}\left[2P_0P_2 + P_1^2\right] + \Gamma A\left[N_0P_2 + P_0N_2 + N_1P_1\right]$$
$$- \Gamma A\hat{\varepsilon}\left[2N_0P_0P_2 + P_0^2N_2 + N_0P_1^2 + 2P_0N_1P_1\right] \qquad (4C)$$

For c^3

$$P_3' = \frac{\beta\Gamma}{\tau_n}N_3 - (\beta\Gamma N_{tr} + \frac{1}{\tau_p})P_3 + \Gamma A\hat{\varepsilon}N_{tr}\left[2P_0P_3 + 2P_1P_2\right] + \Gamma A\left[N_0P_3 + P_0N_3 + N_1P_2 + N_2P_1\right]$$

$$- \Gamma A\hat{\varepsilon}\left[2N_0P_0P_3 + P_0^2N_3 + N_1P_1^2 + 2N_1P_0P_2 + 2N_0P_1P_2 + 2P_0P_1N_2\right] \qquad (4D)$$

Substituting the expansion for N and P in equation (3) we obtain

$$\sum_{n=0}^{\infty} c^n\left[P_n' + N_n'\right] = \frac{I_0}{qV} + c\frac{\hat{I}}{qV} + \frac{1}{\tau_n}(\beta\Gamma - 1)\sum_{n=0}^{\infty} c^n N_n - \frac{1}{\tau_p}\sum_{n=0}^{\infty} c^n P_n \qquad (5)$$

For c^0

$$P_0' + N_0' = \frac{I_0}{qV} + \frac{1}{\tau_n}(\beta\Gamma - 1)N_0 - \frac{1}{\tau_p}P_0 \qquad (5A)$$

For c^1

$$P_1' + N_1' = \frac{\hat{I}}{qV} + \frac{1}{\tau_n}(\beta\Gamma - 1)N_1 - \frac{1}{\tau_p}P_1 \qquad (5B)$$

For c^2

$$P_2' + N_2' = \frac{1}{\tau_n}(\beta\Gamma - 1)N_2 - \frac{1}{\tau_p}P_2 \qquad (5C)$$

For c^3

$$P_3' + N_3' = \frac{1}{\tau_n}(\beta\Gamma - 1)N_3 - \frac{1}{\tau_p}P_3 \qquad (5D)$$

We now solve simultaneous equations (4) and (5) for the Volterra operator N_n and P_n successively.

To solve for N_0 and P_0 from equation (4A) and (5A) we note that $N_0' + P_0' = 0$ since I_0=constant. Thus from equation(4A) we have

$$0 = \frac{\beta\Gamma}{\tau_n}N_0 - (\Gamma A N_{tr} + \frac{1}{\tau_p})P_0 + \Gamma A\hat{\varepsilon}N_{tr}P_0^2 + \Gamma A N_0 P_0 - \Gamma A\hat{\varepsilon}N_0 P_0^2 \qquad (6A)$$

and from equation (5A)

$$0 = \frac{I_0}{qV} + \frac{1}{\tau_n}(\beta\Gamma - 1)N_0 - \frac{1}{\tau_p}P_0 \qquad (6B)$$

solving equation(6B) for P_0 we have

$$P_0 = \frac{\tau_p}{qV}I_0 + \frac{\tau_p}{\tau_n}(\beta\Gamma - 1)N_0 \qquad (6C)$$

now substitute equation (6C) into (6A) to obtain

$$0=\frac{\beta\Gamma}{\tau_n}N_0-(\Gamma AN_{tr}+\frac{1}{\tau_p})\left[\frac{\tau_p}{qV}I_0+\frac{\tau_p}{\tau_n}(\beta\Gamma-1)N_0\right]+\Gamma A\hat{\varepsilon}N_{tr}\left[\frac{\tau_p}{qV}I_0+\frac{\tau_p}{\tau_n}(\beta\Gamma-1)N_0\right]^2$$
$$+\Gamma AN_0\left[\frac{\tau_p}{qV}I_0+\frac{\tau_p}{\tau_n}(\beta\Gamma-1)N_0\right]-\Gamma A\hat{\varepsilon}N_0\left[\frac{\tau_p}{qV}I_0+\frac{\tau_p}{\tau_n}(\beta\Gamma-1)N_0\right]^2$$

We now expand the brackets to obtain

$$0=\frac{\beta\Gamma}{\tau_n}N_0-(\Gamma AN_{tr}+\frac{1}{\tau_p})\frac{\tau_p}{qV}I_0$$
$$-(\Gamma A\varepsilon N_{tr}+\frac{1}{\tau_n})\frac{\tau_p}{\tau_n}(\beta\Gamma-1)N_0$$
$$+\Gamma A\hat{\varepsilon}N_{tr}\left[\frac{\tau_p}{qV}I_0\right]^2+\Gamma A\hat{\varepsilon}N_{tr}\left[\frac{\tau_p}{\tau_n}(\beta\Gamma-1)\right]^2N_0^2$$
$$+2\Gamma A\hat{\varepsilon}N_{tr}\frac{\tau_p}{qV}I_0\frac{\tau_p}{\tau_n}(\beta\Gamma-1)N_0$$
$$+\Gamma A\frac{\tau_p I_0 N_0}{qV}+\left[\frac{\Gamma A\tau_p}{\tau_n}(\beta\Gamma-1)\right]N_0^2$$
$$-\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{qV}I_0\right]^2N_0-\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{\tau_n}(\beta\Gamma-1)\right]^2N_0^3$$
$$-2\Gamma A\hat{\varepsilon}\frac{\tau_p}{qV}I_0\frac{\tau_p}{\tau_n}(\beta\Gamma-1)^2N_0{}^2$$

Collecting terms in like power of N_0 we obtain the cubic:

$$0=-\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{\tau_n}(\beta\Gamma-1)\right]^2N_0^3$$
$$+\left[\Gamma A\hat{\varepsilon}N_{tr}\left[\frac{\tau_p}{\tau_n}(\beta\Gamma-1)\right]^2+\frac{\Gamma A\tau_p}{\tau_n}(\beta\Gamma-1)-2\Gamma A\hat{\varepsilon}\frac{\tau_p}{qV}I_0\frac{\tau_p}{\tau_n}(\beta\Gamma-1)\right]N_0$$
$$+\left[\frac{\beta\Gamma}{\tau_n}-(\Gamma AN_{tr}+\frac{1}{\tau_p})\frac{\tau_p}{\tau_n}(\beta\Gamma-1)+2\Gamma A\hat{\varepsilon}N_0\frac{\tau_p}{qV}I_0\frac{\tau_p}{\tau_n}(\beta\Gamma-1)+\Gamma A\frac{\tau_p}{qV}I_0-\Gamma A\hat{\varepsilon}\left(\frac{\tau_p}{qV}I_0\right)^2\right]N_0+$$
$$\left[-(\Gamma AN_{tr}+\frac{1}{\tau_p})\left(\frac{\tau_p}{qV}I_0\right)+\Gamma A\hat{\varepsilon}N_{tr}\left(\frac{\tau_p}{qV}I_0\right)^2\right]\tag{6D}$$

This a cubic polynomial

$$N_0^3 + C_2 N_0^2 + C_1 N_0 + C_0 = 0 \qquad (7)$$

in which the coefficients are:

$$C_2 = -\frac{\left[\Gamma A \hat{\varepsilon} N_{tr} \left[\frac{\tau_p}{\tau_n} (\beta\Gamma - 1) \right]^2 + \frac{\Gamma A \tau_p}{\tau_n} (\beta\Gamma - 1) - 2\Gamma A \hat{\varepsilon} \frac{\tau_p^2}{qV} I_0 \frac{1}{\tau_n} (\beta\Gamma - 1) \right]}{\Gamma A \hat{\varepsilon} \left[\frac{\tau_p}{\tau_n} (\beta\Gamma - 1) \right]^2} \qquad (7A)$$

$$C_1 = -\frac{\left[\frac{\beta\Gamma}{\tau_n} - \frac{\tau_p}{\tau_n} (\Gamma A N_{tr} + \frac{1}{\tau_p})(\beta\Gamma - 1) + 2\Gamma A \hat{\varepsilon} N_{tr} I_0 \frac{\tau_p^2}{qV\tau_n} (\beta\Gamma - 1) + \Gamma A \frac{\tau_p}{qV} I_0 - \Gamma A \hat{\varepsilon} \left(\frac{\tau_p}{qV} I_0 \right)^2 \right]}{\Gamma A \hat{\varepsilon} \left[\frac{\tau_p}{\tau_n} (\beta\Gamma - 1) \right]^2} \qquad (7B)$$

$$C_0 = -\frac{\left[-(\Gamma A N_{tr} + \frac{1}{\tau_p}) \left(\frac{\tau_p}{qV} I_0 \right) + \Gamma A \hat{\varepsilon} N_{tr} \left(\frac{\tau_p}{qV} I_0 \right)^2 \right]}{\Gamma A \hat{\varepsilon} \left[\frac{\tau_p}{\tau_n} (\beta\Gamma - 1) \right]^2} \qquad (7C)$$

Each of these coefficients is polynomial in I_0.
Thus:

$$C_2 = c_{20} + c_{21} I_0 \qquad (8)$$

in which

$$C_{20} = -\frac{\Gamma A \hat{\varepsilon} N_{tr} \left[\frac{\tau_p}{\tau_n} (\beta\Gamma - 1) \right]^2 + \Gamma A \frac{\tau_p}{\tau_n} (\beta\Gamma - 1)}{\Gamma A \hat{\varepsilon} \left[\frac{\tau_p}{\tau_n} (\beta\Gamma - 1) \right]^2}$$

$$C_{20} = -\frac{1 + \hat{\varepsilon} \frac{\tau_p}{\tau_n} (\beta\Gamma - 1) N_{tr}}{\hat{\varepsilon} \frac{\tau_p}{\tau_n} (\beta\Gamma - 1)} \qquad (8A)$$

and

$$C_{21} = \frac{\frac{2\hat{\varepsilon}\tau_p^2}{qV\tau_n}(\beta\Gamma - 1)\Gamma A}{\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{\tau_n}(\beta\Gamma - 1)\right]^2} \qquad \text{(8B)}$$

$$C_{21} = \frac{2\tau_n}{qV(\beta\Gamma - 1)} \qquad \text{(8B)}$$

Also

$$C_1 = c_{10} + c_{11}I_0 + c_{12}I_0^2 \qquad \text{(9)}$$

in which

$$C_{10} = -\frac{\frac{\beta\Gamma}{\tau_n} - \frac{\tau_p}{\tau_n}\left[\Gamma A N_{tr} + \frac{1}{\tau_p}\right](\beta\Gamma - 1)}{\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{\tau_n}(\beta\Gamma - 1)\right]^2}$$

$$C_{10} = \frac{1 - \left(\Gamma A N_{tr}\tau_p\right)\left(\beta\Gamma - 1\right)}{\Gamma A\hat{\varepsilon}\tau_p\left(\beta\Gamma - 1\right)^2}\frac{\tau_n}{\tau_p} \qquad \text{(9A)}$$

$$C_{11} = -\frac{\frac{2\Gamma A\hat{\varepsilon}N_{tr}\tau_p^2}{qV\tau_n}\left(\beta\Gamma - 1\right)^2 + \frac{\Gamma A\tau_p}{qV\tau_n}}{\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{\tau_n}(\beta\Gamma - 1)\right]^2}$$

$$C_{11} = -\frac{2\hat{\varepsilon}N_{tr}\tau_p}{qV\hat{\varepsilon}}\frac{\left(\beta\Gamma - 1\right) + \tau_n}{\left(\beta\Gamma - 1\right)^2}\frac{\tau_n}{\tau_p} \qquad \text{(9B)}$$

and

$$C_{12} = \frac{\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{qV}\right]^2}{\Gamma A\hat{\varepsilon}\left[\frac{\tau_p}{\tau_n}\left(\beta\Gamma - 1\right)\right]^2}$$

$$C_{12} = \left[\frac{\tau_n}{qV\left(\beta\Gamma - 1\right)}\right]^2 \qquad \text{(9C)}$$

Also

$$C_0 = c_{01} I_0 + c_{02} I_0^2 \tag{10}$$

in which

$$C_{01} = \frac{\dfrac{\tau_p}{qV}\left(\Gamma A N_{tr} + \dfrac{1}{\tau_p}\right)}{\Gamma A \hat{\varepsilon}\left[\dfrac{\tau_p}{\tau_n}(\beta\Gamma - 1)\right]^2}$$

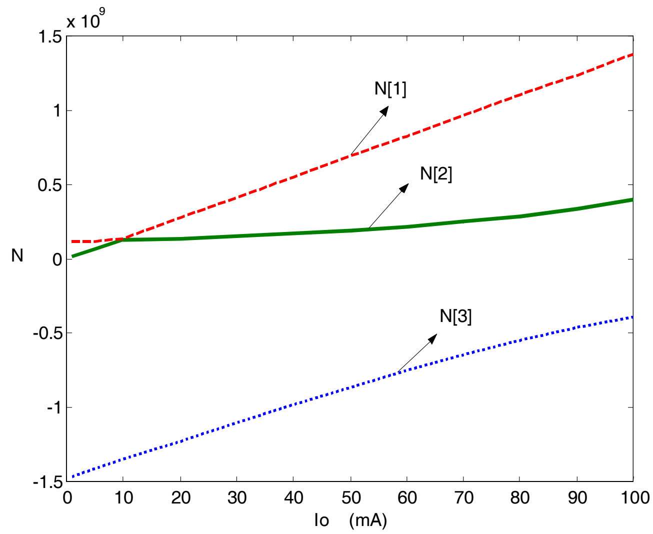
$$C_{01} = \frac{\left(1 + \Gamma A N_{tr} \tau_p\right)\tau_n^2}{qV\Gamma A \hat{\varepsilon} \tau_p^2 \left(\beta\Gamma - 1\right)^2} \tag{10A}$$

and

$$C_{02} = -\frac{\Gamma A \hat{\varepsilon} N_{tr}\left(\dfrac{\tau_p}{qV}\right)^2}{\Gamma A \hat{\varepsilon}\left[\dfrac{\tau_p}{\tau_n}\beta\Gamma - 1\right]^2}$$

$$C_{02} = -\left[\frac{\tau_n}{qV\left(\beta\Gamma - 1\right)}\right]^2 N_{tr}$$

I[0] (mA)	N[1]	P[1]	***N[2]***	***P[2]***	N[3]	P[3]
1	-.1467259972e10	.140006e6	***.13749985e8***	***.3102***	.115969558e9	-.74338e5
5	-.1416062618e10	.1079832e7	***.68747800e8***	***3.099***	.119777690e9	-.37108e5
10	-.1352436817e10	.1083560e7	***.124865398e9***	***.9191e4***	.137538416e9	-24.938
15	-.1289270627e10	.1087623e7	***.130477016e9***	***.55110e5***	.206264734e9	-6.2158
20	-.1226622294e10	.1092062e7	***.136583079e9***	***.100669e6***	.275014463e9	-4.5184
30	-.1103158230e10	.1102272e7	***.150624473e9***	***.190458e6***	.412517255e9	-3.5491
40	-.982715730e9	.1114680e7	***.167686571e9***	***.278049e6***	.550020907e9	-3.2053
50	-.866183936e9	.1129933e7	***.188659144e9***	***.362797e6***	.687524790e9	-3.0291
60	-.754720709e9	.1148871e7	***.214700197e9***	***.443859e6***	.825028767e9	-2.9215
70	-.649776392e9	.1172550e7	***.247260097e9***	***.520179e6***	.962532794e9	-2.8495
80	-.553037478e9	.1202197e7	***.288025379e9***	***.590533e6***	.1100036849e10	-2.7985
90	-.466219398e9	.1239058e7	***.338711478e9***	***.653671e6***	.1237540920e10	-2.7591
100	-.390683388e9	.1284124e7	***.400679635e9***	***.708605e6***	.1375045003e10	-2.728
500	-.18366120e8	.5013356e7	***.5528527641e10***	***.979373e6***	.6875209739e10	-2.533
1000	-.2646556e7	.10001924e8	***.1238801435e11***	***.990805e6***	.1375041598e11.	-2.52
1238	-.3807e4	.12380000e8	***.1565796988e11***	***.992727e6***	.1702301405e11	-2.44
1239	.4970e4	***.12389996e8***	***.1567171143e11***	***.992733e6***	.1703676455e11	-2.51
1300	.512759e7	***.12999627e8***	***.1650997879e11***	***.993102e6***	.1787553974e11	-2.50
1500	.1867543e7	***.14998641e8***	***.1925870654e11***	***.994088e6***	.2062562220e11	-2.50



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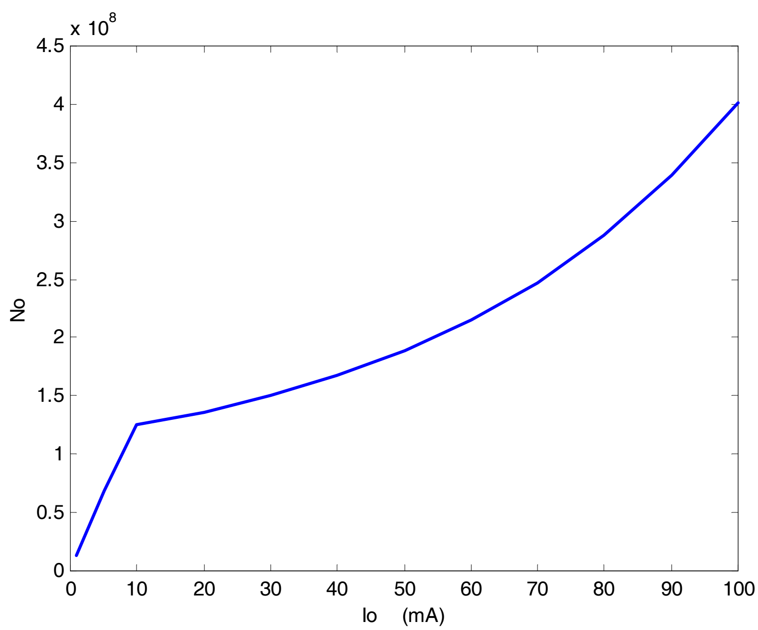
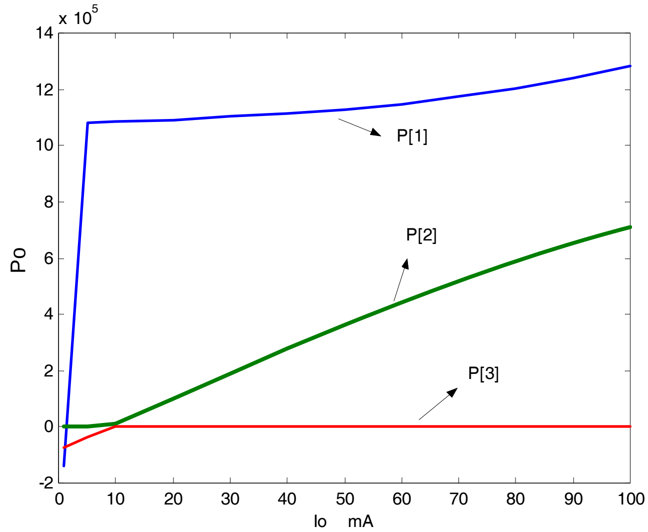
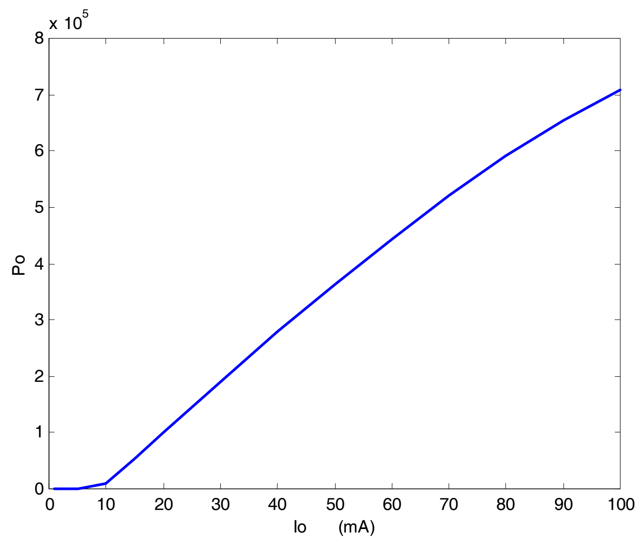




There are three roots for N_0 of Equation (7). For each root, there is a corresponding value of P_0 obtained from equation (6C).

$$P_0 = \frac{\tau_p}{qV} I_0 + \frac{\tau_p}{\tau_n}(\beta\Gamma - 1)N_0 \qquad (6C)$$



for a physical solution, we require $N_0>0$ and $P_0>0$ .We thus expect that only one of these solution pairs satisfy this condition for any value of I_0. The procedure to determine N_0 and P_0 is to use equation(8), (9) and (10) to determine the coefficients C_0, C_1 and C_2 as polynomials in I_0 and solving equation (7) and (6C) for the three solution pairs of N_0 and P_0.IF our equations are physically proper ones, there will be only one pair for which both N_0 and P_0 are positive and real. This is then the physical electron density N_0, and photon density P_0 of the laser diode as a function of the injection current, I_0..

Observe that for large values of I_0

$$C_2 \approx c_{21} I_0 = \frac{2\tau_n I_0}{qV(\beta\Gamma - 1)}$$

$$C_1 \approx c_{12} I_0^2 = \left(\frac{\tau_n}{qV(\beta\Gamma - 1)} \right)^2 I_0^2$$

$$C_0 \approx c_{02} I_0^2 = -\left(\frac{\tau_n}{qV(\beta\Gamma - 1)} \right)^2 N_{tr} I_0^2$$

For convenience, let

$\alpha = \dfrac{\tau_n}{qV(\beta\Gamma - 1)}$. Then

$$C_2 \approx 2\alpha I_0$$

$$C_1 \approx \alpha^2 I_0^2$$

$$C_0 \approx -\alpha^2 I_0^2 N_{tr}$$

The cubic equation for N_0, , equation(7), then is approximately

$$N_0^3 + 2\alpha N_0^2 I_0 + \alpha^2 I_0^2 N_0 - \alpha^2 I_0^2 N_{tr} = 0$$

For large values of I_0 one root of this equation is approximately obtained as

$$\alpha^2 I_0^2 N_0 - \alpha^2 I_0^2 N_{tr} = 0$$

For which the solution is

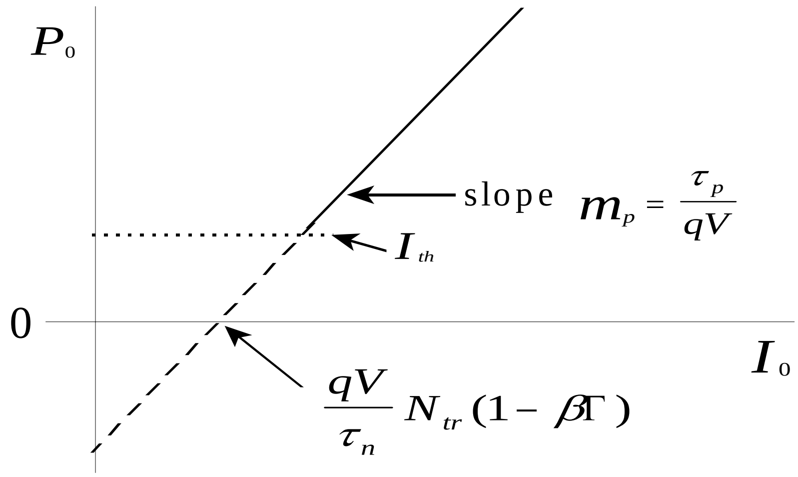
$$N_0 = N_{tr}$$

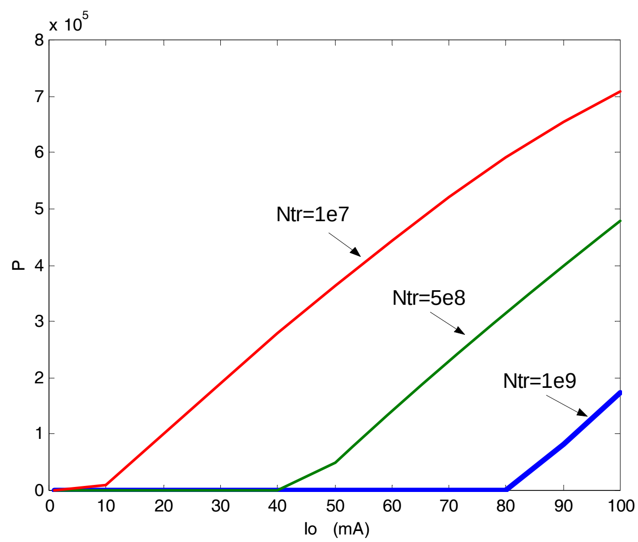
We thus observe that, for large values of I_0 , N_0 is a constant equal to N_{tr} . from a computer solution of equation (7) , this is observed to be true for $I_0 \geq$ 7ma.

We this approximation, the photon density for large ($I_0 \geq$ 7ma) is from equation (6C)

$$P_0 \approx \frac{\tau_p}{qV} I_0 + \frac{\tau_p}{\tau_n} (\beta\Gamma - 1) N_{tr}$$

This the equation of a straight line as show





The slope of straight line is Figure

$$m_p = \frac{\tau_p}{qV}$$

Also, P_0=0 for

$$\frac{\tau_p}{qV} I_0 = \frac{\tau_p}{\tau_n}(1-\beta\Gamma)N_{tr}$$

$$I_0 = \frac{qV}{\tau_n}(1-\beta\Gamma)N_{tr} ....amperes$$

this result is observed valid from an exact computer solution of the equation for $P_0 \geq 0$ note that we thus here that the threshold current $=I_{th}$

$$I_{th} = \frac{qV}{\tau_n}(1-\beta\Gamma)N_{tr}\,. \qquad (12)$$

This corresponds to the numerical value obtained experimentally. Note from our theoretical solution the threshold current, I_{th} is proportional to N_{tr} which is temperature dependent and increase with increasing temperature. However, the slope of $P_0 - I_0$ curves is $m_p = \tau_p / (qV)$ which is essentially independent of temperature. We thus expect the graph of $P_0 - I_0$ curves to be

We now determine the first-order Volterra operator for N_1 and P_1 by solving equation(4B) and (5B) simultaneously

$$P_1' = \frac{\beta\Gamma}{\tau_n} N_1 - (A\Gamma N_{tr} + \frac{1}{\tau_p})P_1 + 2\Gamma A\hat{\varepsilon}N_{tr}P_0P_1 + \Gamma A\left[N_0P_1 + N_1P_0\right]$$
$$-\Gamma A\hat{\varepsilon}\left[2N_0P_0P_1 + P_0^2N_1\right] \qquad (4B)$$

$$P_1' + N_1' = \frac{1}{qV}\hat{I} + \frac{1}{\tau_n}(\beta\Gamma - 1)N_1 - \frac{1}{\tau_p}P_1 \qquad (5C)$$

Collecting terms in equation(4B)

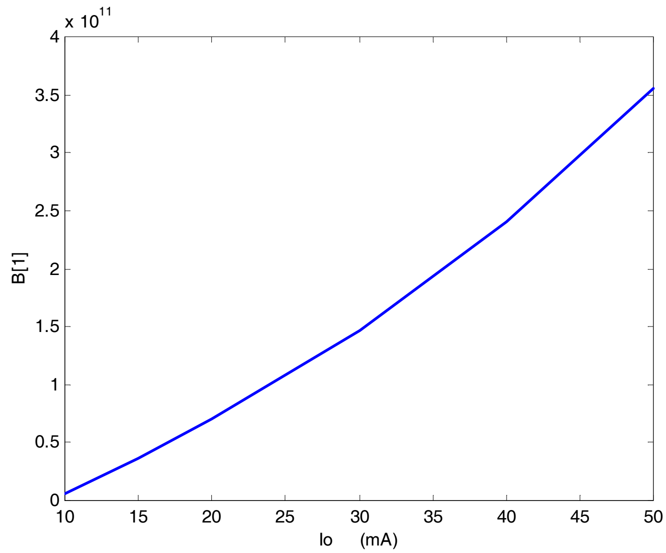
$$P_1' + \left[\Gamma AN_{tr} + \frac{1}{\tau_p} - 2\Gamma A\hat{\varepsilon}N_{tr}P_0 - \Gamma AN_0 + 2\Gamma A\hat{\varepsilon}N_0P_0\right]P_1 = \left[\frac{\beta\Gamma}{\tau_n} + \Gamma AP_0 - \Gamma A\hat{\varepsilon}P_0^2\right]N_1 \qquad (20)$$

This equation can be expressed as

$$P_1' + B_1P_1 = B_0N_1 \qquad (21)$$

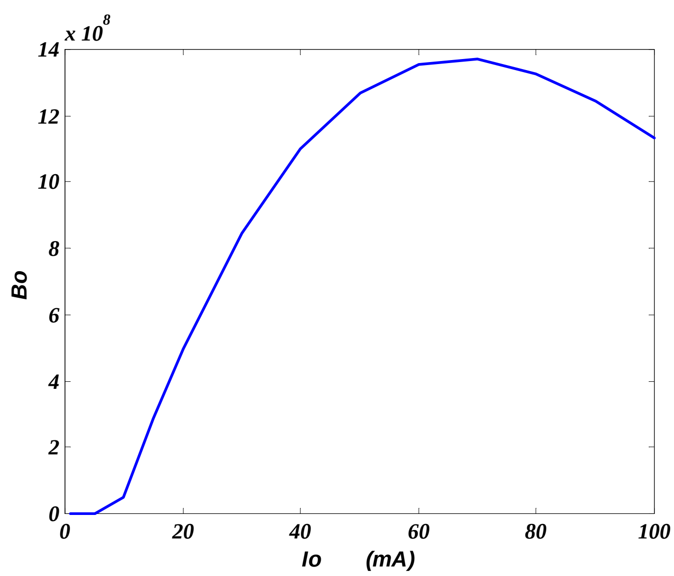
in which

$$B_1 = \Gamma A N_{tr} + \frac{1}{\tau_p} - 2\Gamma A\hat{\varepsilon} N_{tr} P_0 - \Gamma A N_0 + 2\Gamma A\hat{\varepsilon} N_0 P_0 \tag{21A}$$



and

$$B_0 = \frac{\beta\Gamma}{\tau_n} + \Gamma A P_0 - \Gamma A\hat{\varepsilon} P_0^2 \tag{21B}$$



Equation (21) can be solved for N_1 as

$$N_1 = \frac{1}{B_0} P_1' + \frac{B_1}{B_0} P_1 \qquad (21C)$$

We now substitute equation (21C) into equation (5B)

$$P_1' + \frac{1}{B_0} P_1'' + \frac{B_1}{B_0} P_1' = \frac{1}{qV} \hat{I} + \frac{1}{\tau_n} \left(\frac{\beta\Gamma - 1}{B_0} \right) \left[P_1' + B_1 P_1 \right] - \frac{1}{\tau_p} P_1$$

collecting terms

$$\frac{1}{B_0} P_1'' + \left(1 + \frac{B_1}{B_0} - \frac{1}{\tau_n} \frac{\beta\Gamma - 1}{B_0} \right) P_1' + \left(\frac{1}{\tau_p} - \frac{1}{\tau_n} \frac{\beta\Gamma - 1}{B_0} \right) P_1 = \frac{1}{qV} \hat{I}$$

multiplying by B_0

$$P_1'' + \left(B_0 + B_1 - \frac{\beta\Gamma - 1}{\tau_n} \right) P_1' + \left(\frac{B_0}{\tau_p} - \frac{(\beta\Gamma - 1) B_1}{\tau_n} \right) P_1 = \frac{B_0}{qV} \hat{I} \qquad (22)$$

Equation (22) is a linear differential equation

$$P_1'' + D_1 P_1' + D_0 P_1 = \frac{B_0}{qV} \hat{I} \qquad (23)$$

In which

$$D_1 = B_0 + B_1 - \left(\frac{\beta\Gamma - 1}{\tau_n} \right) \qquad (23A)$$

and

$$D_0 = \frac{B_0}{\tau_p} - \left(\frac{\beta\Gamma - 1}{\tau_n} \right) B_1 \qquad (23B)$$

The Laplace transform of this equation is

$$\left[s^2 + D_1 s + D_0 \right] P_1(s) = \frac{B_0}{qV} \hat{I}(s)$$

or

$$P_1(s) = \frac{B_0}{qV} \left(\frac{1}{s^2 + D_1 s + D_0} \right) \hat{I}(s)$$

Then

$$P_1(s) = \frac{B_0}{qV} H_1(s)\hat{I}(s)$$

in which

$$H_1(s) = \frac{1}{s^2 + D_1 s + D_0}$$

which can be represented as

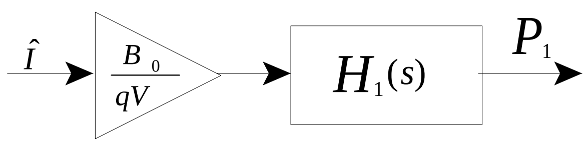
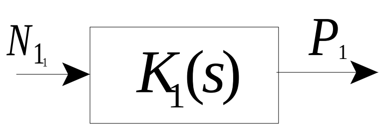


Figure
Now the Laplace transform of equation (21C) is

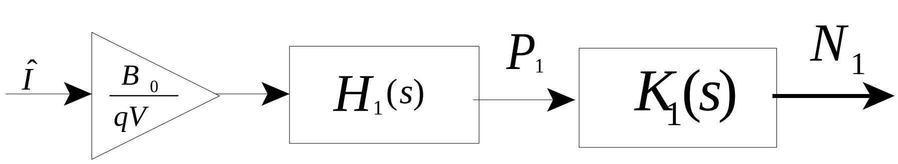
$$N_1(s) = \frac{1}{B_0}\left[s + B_1\right]P_1(s)$$

$$N_1(s) = K_1(s)P_1(s)$$



Figure

We thus have



Figure

$$H_1(s)K_1(s) = \frac{1}{qV}\frac{s + B_1}{s + D_1 s + D_0}$$

So also

Sekil

$$\hat{I} \longrightarrow \boxed{\frac{1}{qV}\frac{s + B_1}{s^2 + D_1 s + D_0}} \xrightarrow{P_1} \boxed{\frac{B_0}{s + B_1}} \xrightarrow{N_1}$$

Figure

We now examine the constants D_0 and D_1 substituting equation (21B) into equation (23A), we obtain

$$D_1 = \Gamma A N_{tr} + \frac{1}{\tau_p} - 2\Gamma A \hat{\varepsilon} N_{tr} P_0 - \Gamma A N_0 + 2\Gamma A \hat{\varepsilon} N_0 P_0 + \frac{\beta\Gamma}{\tau_n} + \Gamma A P_0 - \Gamma A \hat{\varepsilon} P_0^2 - \frac{\beta\Gamma - 1}{\tau_n}$$

Collecting terms we have obtain

$$D_1 = \Gamma A\left(N_{tr} - N_0\right) + 2\Gamma A \hat{\varepsilon} P_0\left(N_0 - N_{tr}\right) + \Gamma A\left(P_0 - \hat{\varepsilon} P_0^2\right) + \frac{1}{\tau_n} + \frac{1}{\tau_p}$$

$$D_1 = \Gamma A\left(1 - 2\hat{\varepsilon} P_0\right)\left(N_{tr} - N_0\right) + \Gamma A P_0\left(1 - \hat{\varepsilon} P_0^2\right) + \frac{1}{\tau_n} + \frac{1}{\tau_p} \qquad (24A)$$

Similarly for D_0 we substitute equation (21A) and (21B) into equation (23B) to obtain

$$D_0 = \frac{\beta\Gamma}{\tau_n \tau_p} + \frac{\Gamma A}{\tau_p} P_0 - \frac{\Gamma A \hat{\varepsilon}}{\tau_p} P_0^2 - \frac{\beta\Gamma - 1}{\tau_n} \Gamma A N_{tr}$$

$$- \frac{\beta\Gamma - 1}{\tau_n \tau_p} + \frac{2\Gamma A \hat{\varepsilon} N_{tr}\left(\beta\Gamma - 1\right)}{\tau_n} P_0 \qquad (24A)$$

$$+ \frac{\beta\Gamma - 1}{\tau_n} \Gamma A N_0 - \frac{2\Gamma A \hat{\varepsilon} N_0 P_0 \left(\beta\Gamma - 1\right)}{\tau_n}$$

collecting terms,

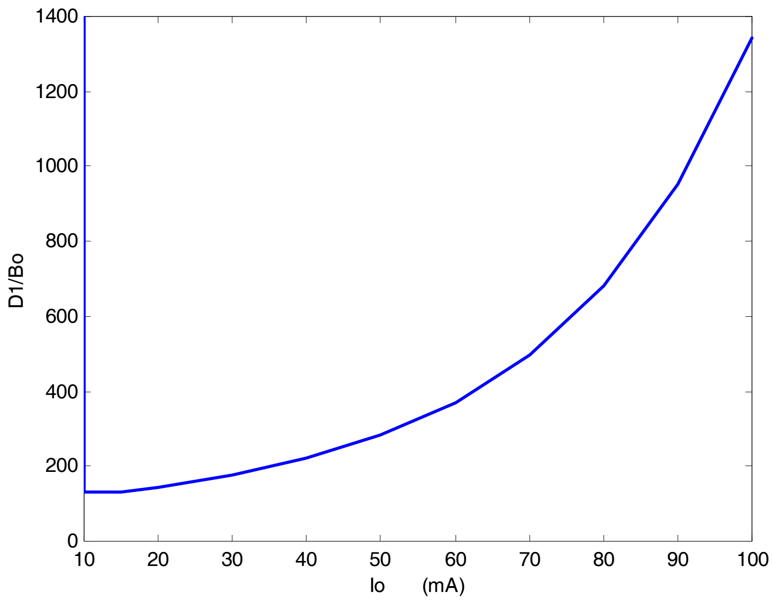
$$D_0 = \frac{\beta\Gamma - 1}{\tau_n}\Gamma A\left[N_0 - N_{tr}\right] + \frac{2\Gamma A\hat{\varepsilon}(\beta\Gamma - 1)P_0}{\tau_n}\left[N_{tr} - N_0\right] + \frac{\Gamma A}{\tau_p}P_0 - \frac{\Gamma A\hat{\varepsilon}}{\tau_p}P_0^2 + \frac{1}{\tau_n\tau_p}$$

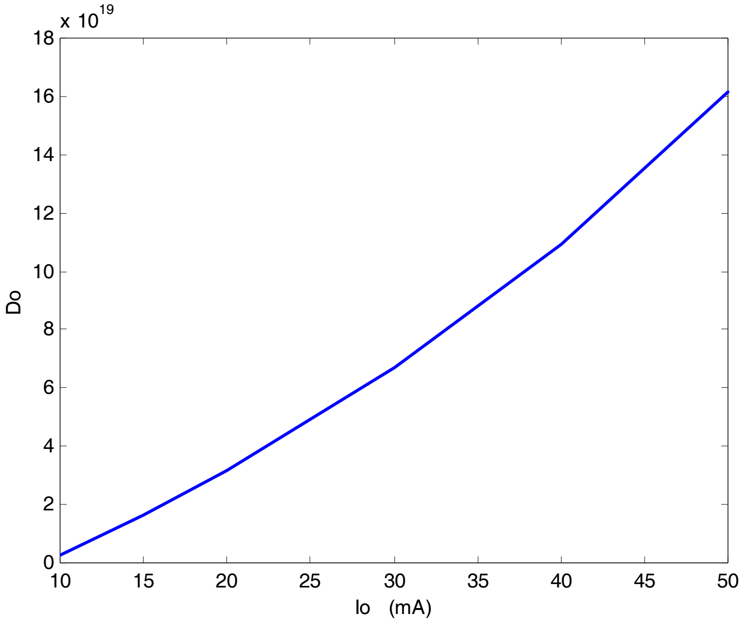
$$D_0 = \frac{\beta\Gamma - 1}{\tau_n}\Gamma A(1 - 2\hat{\varepsilon}P_0)\left[N_0 - N_{tr}\right] + \frac{\Gamma A}{\tau_p}P_0\left(1 - \hat{\varepsilon}P_0^2\right) + \frac{1}{\tau_n\tau_p} \tag{24B}$$

We now observe that for large I_0, $N_0 = N_{tr}$ Thus for large I_0

$$D_1 = \Gamma A P_0(1 - \hat{\varepsilon}P_0) + \frac{1}{\tau_n} + \frac{1}{\tau_p} \tag{25A}$$

$$D_0 = \frac{\Gamma A}{\tau_p}P_0(1 - \hat{\varepsilon}P_0) + \frac{1}{\tau_n}\frac{1}{\tau_p} \tag{25B}$$





The poles of $H_1(s)$ are the roots of

$$s^2 + D_1 s + D_0 = \left(s + \frac{D_1}{2}\right)^2 - \left(\frac{D_1^2}{4} - D_0\right)$$

$$= s + \frac{D_1}{2} \pm \sqrt{\frac{D_1^2}{4} - D_0}$$

so the roots are

$$s_1 = -\frac{D_1}{2} + \sqrt{\left(\frac{D_1}{2}\right)^2 - D_0}$$

$$s_2 = -\frac{D_1}{2} - \sqrt{\left(\frac{D_1}{2}\right)^2 - D_0}$$

We thus here

$$s_{1,2} = -\frac{D_1}{2} + \left[1 \pm \sqrt{1 - \frac{4D_0}{D_1^2}}\right]$$

now

$\frac{4D_0}{D_1^2} < 10^{-18}$ so that

$$\sqrt{1-\frac{4D_0}{D_1^2}} \approx 1-\frac{2D_0}{D_1^2}$$

Consequently

$$s_{1,2} = -\frac{D_1}{2}\left[1 \pm \left(1-\frac{2D_0}{D_1^2}\right)\right]$$

or

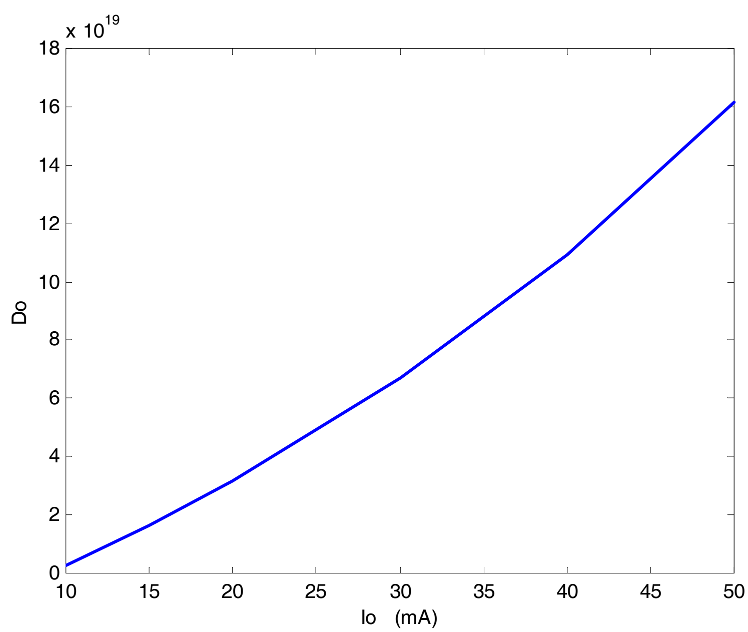
$$s_1 \approx -D_1$$

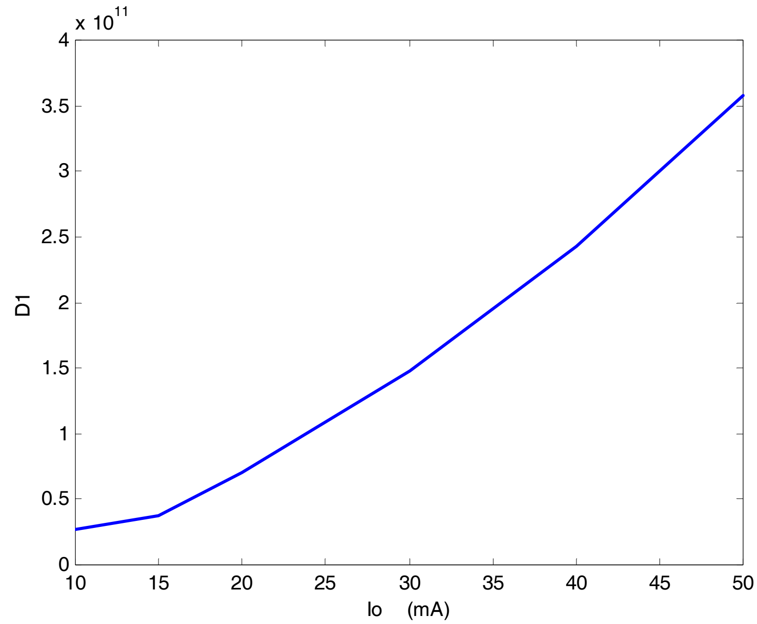
$$s_2 = -\frac{D_1}{2}\left(\frac{2D_0}{D_1^2}\right) = -\frac{D_0}{D_1}$$

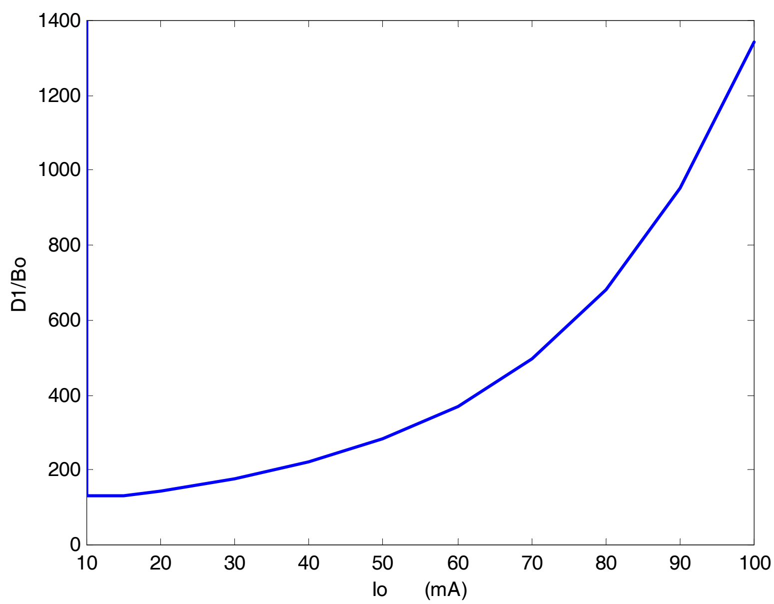
note that if $1-2\hat{\varepsilon}P_0 \approx 1$, we would have
$D_1 \approx \Gamma A P_0$ and $D_1 \approx \left(\Gamma A P_0\right)/\tau_p$ so that

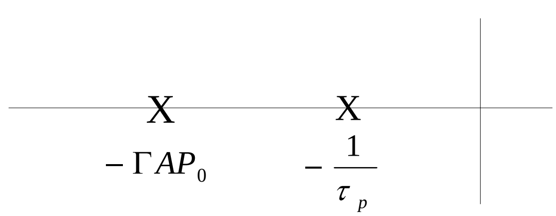
$$s_1 \approx -\Gamma A P_0$$

$$s_2 \approx -\frac{1}{\tau_p}$$




x 10^11
D1
Io (mA)
0
0.5
1
1.5
2
2.5
3
3.5
4
10
15
20
25
30
35
40
45
50


D1/Bo
Io (mA)
0
200
400
600
800
1000
1200
1400
10
20
30
40
50
60
70
80
90
100


X
X
$-\Gamma A P_0$
$-\frac{1}{\tau_p}$

Figure

Note that the system is stable. Also, are pole is proportional the P_0 which is proportional to I_0 .

We now determine the second-order Volterra operator, P_2 and N_2 .For this We use equation(4C) and (5C)

$$P_2' = \frac{\beta\Gamma}{\tau_n} N_2 - (\beta\Gamma N_{tr} + \frac{1}{\tau_p}) P_2 + \Gamma A\hat{\varepsilon} N_{tr} \left[2P_0P_2 + P_1^2 \right] + \Gamma A \left[N_0P_2 + P_0N_2 + N_1P_1 \right]$$
$$- \Gamma A\hat{\varepsilon} \left[2N_0P_0P_2 + P_0^2N_2 + N_0P_1^2 + 2P_0N_1P_1 \right] \qquad (4C)$$

$$P_2' + N_2' = \frac{1}{\tau_n}(\beta\Gamma - 1)N_2 - \frac{1}{\tau_p}P_2 \qquad (5C)$$

Collecting terms in equation(4C)

$$P_2' + \left[\Gamma A N_{tr} + \frac{1}{\tau_p} - 2\Gamma A\hat{\varepsilon} N_{tr} P_0 - \Gamma A\hat{\varepsilon} N_0 + 2\Gamma A\hat{\varepsilon} N_0 P_0 \right] P_2$$
$$= \left(\frac{\beta\Gamma}{\tau_n} + \Gamma A P_0 - \Gamma A\hat{\varepsilon} P_0^2 \right) N_2 + \left(\Gamma A\hat{\varepsilon} N_{tr} P_1^2 + \Gamma A N_1 P_1 - \Gamma A\hat{\varepsilon} N_0 P_1^2 - 2\Gamma A\hat{\varepsilon} P_0 N_1 P_1 \right) \qquad (40)$$

This equation can be expressed as

$$P_2' + B_1 P_2 = B_0 N_2 + F_2 \qquad (41)$$

in which B_0 and B_1 are given by equation (21B) and (21A) respectively and

$$F_2 = \Gamma A\hat{\varepsilon} N_{tr} P_1^2 + \Gamma A N_1 P_1 - \Gamma A\hat{\varepsilon} N_0 P_1^2 - 2\Gamma A\hat{\varepsilon} P_0 N_1 P_1$$

$$F_2 = \Gamma A N_1 P_1 \left[1 - 2\hat{\varepsilon} P_0 \right] - \Gamma A\hat{\varepsilon} P_1^2 \left[N_0 - N_{tr} \right] \qquad (41A)$$

Equation 41 can be solved for N_2 as

$$N_2 = \frac{1}{B_0} P_2' + \frac{B_1}{B_0} P_2 - \frac{1}{B_0} F_2 \qquad (41B)$$

We now substitute equation (41B) in equation (5C)

$$P_2' + \frac{1}{B_0} P_2'' + \frac{B_1}{B_0} P_2' - \frac{1}{B_0} F_2' = \frac{(\beta\Gamma - 1)}{\tau_n} \frac{1}{B_0} P_2' + \frac{(\beta\Gamma - 1)}{\tau_n} \frac{B_1}{B_0} P_2 - \frac{(\beta\Gamma - 1)}{\tau_n B_0} F_2 - \frac{1}{\tau_p} P_2$$

Collecting terms

$$\frac{1}{B_0}P_2''+\left[1+\frac{B_1}{B_0}-\frac{(\beta\Gamma-1)}{\tau_n}\frac{1}{B_0}\right]P_2'+\left[\frac{1}{\tau_p}-\frac{(\beta\Gamma-1)}{\tau_n}\frac{B_1}{B_0}\right]P_2=\frac{1}{B_0}F_2'-\frac{(\beta\Gamma-1)}{\tau_n B_0}F_2$$

Multiplying by B_0

$$P_2''+\left[B_0+B_1-\frac{(\beta\Gamma-1)}{\tau_n}\right]P_2'+\left[\frac{B_0}{\tau_p}-\frac{(\beta\Gamma-1)}{\tau_n}B_1\right]P_2=F_2'-\frac{(\beta\Gamma-1)}{\tau_n}F_2 \qquad (42)$$

This equation is a linear differential equation equation

$$P_2''+D_1P_2'+D_0P_2=F_2'-\frac{(\beta\Gamma-1)}{\tau_n}F_2 \qquad (43)$$

In which D_1 and D_0 are given by equation (23A) and (23B) respectively. A block diagram thus in

$$F_2'-\frac{\beta\Gamma-1}{\tau_n}F_2 \longrightarrow \boxed{H_1(s)} \xrightarrow{P_2}$$

Figure

Now from equation (41B)

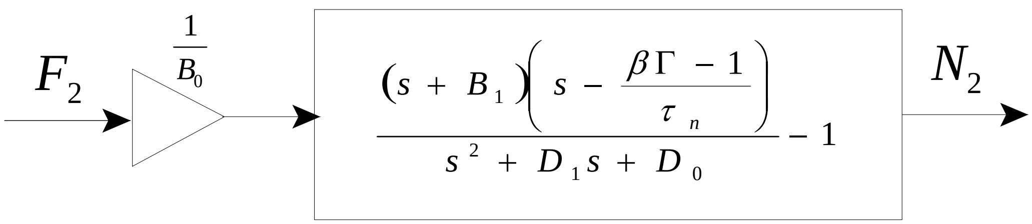
$$N_2(s)=\frac{s+B_1}{B_0}P_2(s)-\frac{1}{B_0}F_2(s)$$

and from equation (43)

$$P_2(s)=\frac{s-\dfrac{\beta\Gamma-1}{\tau_n}}{s^2+D_1s+D_0}F_2(s)$$

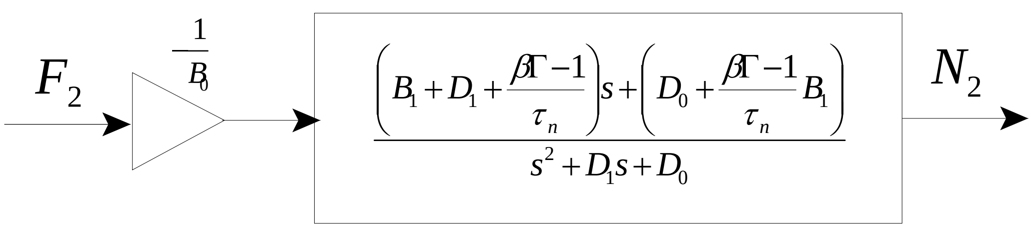
so

$$N_2(s)=\frac{1}{B_0}\left[\frac{(s+B_1)\left(s-\frac{\beta\Gamma-1}{\tau_n}\right)}{s^2+D_1s+D_0}-1\right]F_2(s)$$



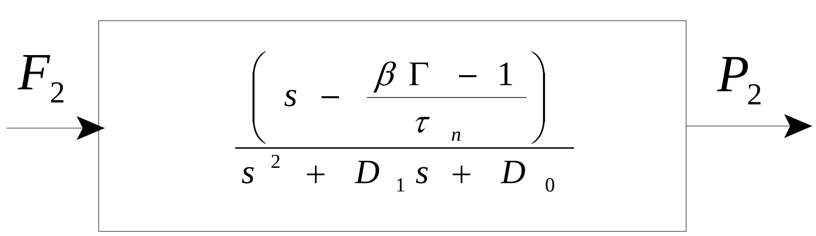
Figure

Which is equivalent to



Figure

and



Figure

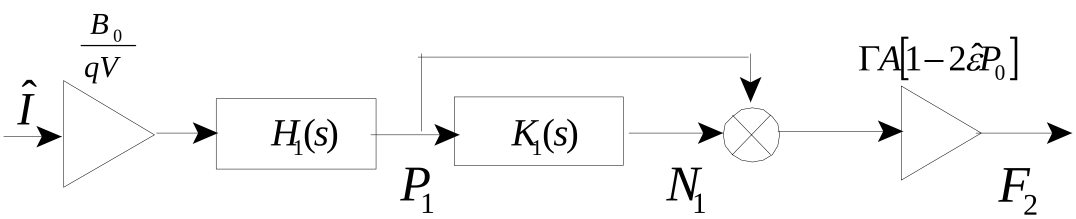
Now from Equation (41A)

$$F_2 = \Gamma A N_1 P_1 \left[1 - 2\hat{\varepsilon} P_0\right] - \Gamma A \hat{\varepsilon} P_1^2 \left[N_0 - N_{tr}\right]$$

For large I_0, $N_0 = N_{tr}$ so that

$$F_2 = \Gamma A N_1 P_1 \left[1 - 2\hat{\varepsilon} P_0\right]$$

which is obtained as



Figure

We now determine the third-order Volterra operators, P_3 and N_3. For this, We use equation (4D) and (5D)

$$P_3' = \frac{\beta\Gamma}{\tau_n} N_3 - (A\Gamma N_{tr} + \frac{1}{\tau_p}) P_3 + \Gamma A \hat{\varepsilon} N_{tr} \left[2P_0 P_3 + 2P_1 P_2\right] + \Gamma A \left[N_0 P_3 + P_0 N_3 + N_1 P_2 + N_2 P_1\right]$$

$$- \Gamma A \hat{\varepsilon} \left[2N_0 P_0 P_3 + P_0^2 N_3 + N_1 P_1^2 + 2N_1 P_0 P_2 + 2N_0 P_1 P_2 + 2P_0 P_1 N_2\right] \tag{4D}$$

$$P_3' + N_3' = \frac{1}{\tau_n}(\beta\Gamma - 1) N_3 - \frac{1}{\tau_p} P_3 \tag{5D}$$

Collecting terms in equation (4D)

$$P_3' + \left[\Gamma A N_{tr} + \frac{1}{\tau_p} - 2\Gamma A \hat{\varepsilon} N_{tr} P_0 - \Gamma A N_0 + 2\Gamma A \hat{\varepsilon} N_0 P_0\right] P_3 = \left[\frac{\beta\Gamma}{\tau_p} + \Gamma A P_0 - \Gamma A \hat{\varepsilon} P_0^2\right] N_3$$
$$+ \left[2\Gamma A \hat{\varepsilon} N_{tr} P_1 P_2 + \Gamma A N_1 P_2 + \Gamma A N_2 P_1 - \Gamma A \hat{\varepsilon} N_1 P_1^2 - 2\Gamma A \hat{\varepsilon} N_1 P_0 P_2 - 2\Gamma A \hat{\varepsilon} N_0 P_1 P_2 - 2\Gamma A \hat{\varepsilon} N_2 P_0 P_1\right] \tag{60}$$

This equation can be expressed as

$$P_3' + B_1 P_3 = B_0 N_3 + F_3 \tag{61}$$

in which B_0 and B_1 are given by equation (21B) and (21A) respectively and

$$F_3 = \left[2\Gamma A\hat{\varepsilon}N_{tr}P_1P_2 + \Gamma AN_1P_2 + \Gamma AN_2P_1 - \Gamma A\hat{\varepsilon}N_1P_1^2 - 2\Gamma A\hat{\varepsilon}N_1P_0P_2 - 2\Gamma A\hat{\varepsilon}N_0P_1P_2 - 2\Gamma A\hat{\varepsilon}N_2P_0P_1\right]$$

$$= 2\Gamma A\hat{\varepsilon}P_1P_2\left[N_{tr} - N_0\right] + \Gamma AN_1P_2\left[1 - 2\hat{\varepsilon}P_0\right] + \Gamma AN_2P_1\left[1 - 2\hat{\varepsilon}P_0\right] - \Gamma A\hat{\varepsilon}N_1P_1^2$$

Equation (61) can be solved for N_3 as

$$N_3 = \frac{1}{B_0}P_3' + \frac{B_1}{B_0}P_3 - \frac{1}{B_0}F_3 \qquad (61B)$$

We now substitute equation (61B) in equation (5D)

$$P_3' + \frac{1}{B_0}P_3'' + \frac{B_1}{B_0}P_3' + \frac{1}{B_0}F_3' = \frac{(\beta\Gamma - 1)}{\tau_n}\frac{1}{B_0}P_3' + \frac{(\beta\Gamma - 1)}{\tau_n}\frac{B_1}{B_0}P_3' - \frac{(\beta\Gamma - 1)}{\tau_n}\frac{1}{B_0}F_3' - \frac{1}{\tau_p}P_3$$

Collecting terms and multiplying by B_0 we obtain

$$P_3'' + \left[B_0 + B_1 - \frac{(\beta\Gamma - 1)}{\tau_n}\right]P_3' + \left[\frac{B_0}{\tau_p} - \frac{(\beta\Gamma - 1)}{\tau_n}B_1\right]P_3 = F_3' + \frac{(\beta\Gamma - 1)}{\tau_n}F_3 \qquad (62)$$

This is a linear differential equation

$$P_3'' + D_1P_3' + D_0P_3 = F_3' + \frac{(\beta\Gamma - 1)}{\tau_n}F_3 \qquad (63)$$

In which D_1 and D_0 are given by equation (23A) and (23B) respectively.
Similar to second-order operator, we have

$$F_3 \longrightarrow \boxed{\frac{\left(s - \dfrac{\beta\Gamma - 1}{\tau_n}\right)}{s^2 + D_1 s + D_2}} \longrightarrow P_3$$

Figure

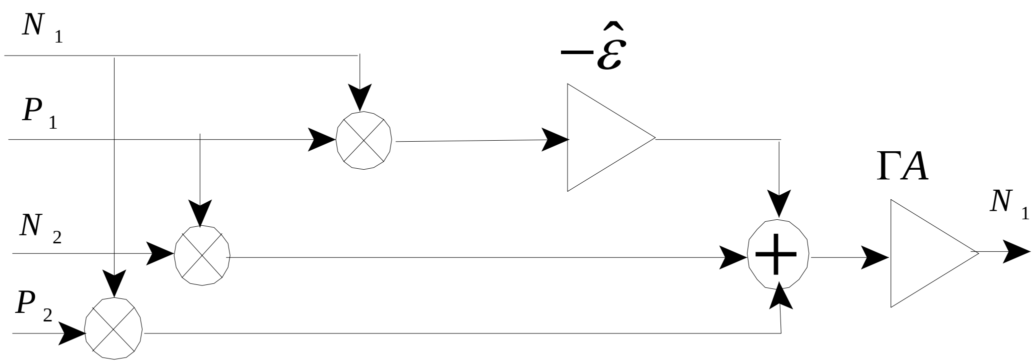
In which F_3 is given by equation (61A) note that N_0=N_{tr} for large Io so that for large I0

$$F_3 = \Gamma AN_1P_2\left[1 - 2\hat{\varepsilon}P_0\right] + \Gamma AN_2P_1\left[1 - 2\hat{\varepsilon}P_0\right] - \Gamma A\hat{\varepsilon}N_1P_1^2$$

Also, for $1-2\hat{\varepsilon}P_0 \approx 1$ we have

$$F_3 = \Gamma A N_1 P_2 + \Gamma A N_2 P_1 - \Gamma A \hat{\varepsilon} N_1 P_1^2 = \Gamma A\left(N_1 P_2 + N_2 P_1 - \hat{\varepsilon} N_1 P_1^2\right)$$

which is obtained as



Figure

In which N_1, P_1, N_2, and P_2 are fist and second-order operator of n and P respectively. Also N_3, if desired, is obtained from equation (61B).

For linear feedback in which K=K,

$$P_1 = H_1 R_1 \tag{116}$$

$$P_2 = R_1 H_2 R_1 \tag{117}$$

$$P_3 = R_1 H_3 R_1 - 2R_1 H_2\{I, R_1, K_1, H_2\}R_1$$

$$P_3 = R_1 H_3 R_1 + R_1 H_2[I - R_1 K_1 H_2]R_1 - R_1 H_2 R_1 K_1 H_2 R_1 - R_1 H_2 R_1 \tag{118}$$

$$P_3 = R_1[H_3 + H_2[I - R_1 K_1 H_2] - H_2 R_1 K_1 H_2 - H_2]R_1$$

sec too #7 for linearization using NL feedback in our case,

$$K_1 = K e^{-st_0} = K e^{-s_n} \tag{119}$$

note that feedback system operator ,P_n are stable operators if R_1 is a stable operator now

$$R_1(s) = \frac{1}{1 + H_1(s) K_1(s)} \tag{120}$$

$$R_1(s) = \frac{1}{1 + \frac{B_0 K}{q} \frac{e^{-st_0}}{s^2 + D_1 s + D_0}} \tag{121}$$

$$R_1(s) = \frac{s^2 + D_1 s + D_0}{s^2 + D_1 s + D_0 + \frac{B_0 K}{q} e^{-st_0}} \tag{122}$$

The eq (27) is a stable operator if the roots of

$$F(s) = s^2 + D_1 s + D_0 + \frac{B_0 K}{q} e^{-t_0 s} \tag{123}$$

are all in the left- half plane.

For convenience, we define $s_n = t_0 s = \sigma_n + j\omega_n$ in which $\sigma_n = t_0\sigma$ and $\omega_n = t_0\omega$. Then

$$F(s_n) = s_n{}^2 + D_1 t_0 s_n + D_0 t_0^2 + \frac{B_0 K t_0}{q} e^{-s_n} \cos\omega_n \tag{124}$$

Then

$$F_r(\sigma_n, \omega_n) = \sigma_n{}^2 - \omega_n^2 + D_1 t_0 \sigma_n + D_0 t_0^2 + \frac{B_0 K t_0^2}{q} e^{-s_n} \cos\omega_n \tag{125}$$

$$F_i(\sigma_n, \omega_n) = 2\sigma_n - \omega_n + D_1 t_0 \sigma_n - \frac{B_0 K t_0^2}{q} e^{-s_n} \sin\omega_n \tag{126}$$

$F_i(\sigma_n, \omega_n)$ in which $F(s_n) = F_r(\sigma_n, \omega_n) + jF_i(\sigma_n, \omega_n)$. Now F=0 if only if F_r and F_i=0. From eq.(125) $F_i(\sigma_n, \omega_n) = 0$

$$\frac{Sin\omega_n}{\omega_n} = \frac{q}{B_0 K t_0^2}\left[D_1 t_0 + 2\sigma_n\right] e^{+\sigma_n} \tag{127}$$

To the equation to be stable, this equation can not have any solutions for $\sigma_n \geq 0$. Now the right side of eq. (127) is a monotonically incresing function of σ_n. Thus, eq(127) has no solutions for $\sigma_n \geq 0$ if

$$\frac{qD_1 t_0}{B_0 K t_0^2} = \frac{qD_1}{B_0 K t_0} > 1 \tag{128}$$

or

$$K < \frac{qD_1}{B_0 t_0} = K_0 \tag{129}$$

The range of K which the system is stable can be large since we have not considered that eq(125) must be simultaneously satisfied. For our laser diode parameters, result of a computer simulate are

t_0	10^{-12}	10^{-11}	10^{-10}
K_{max}/K_0	1.01	1.26	1.32

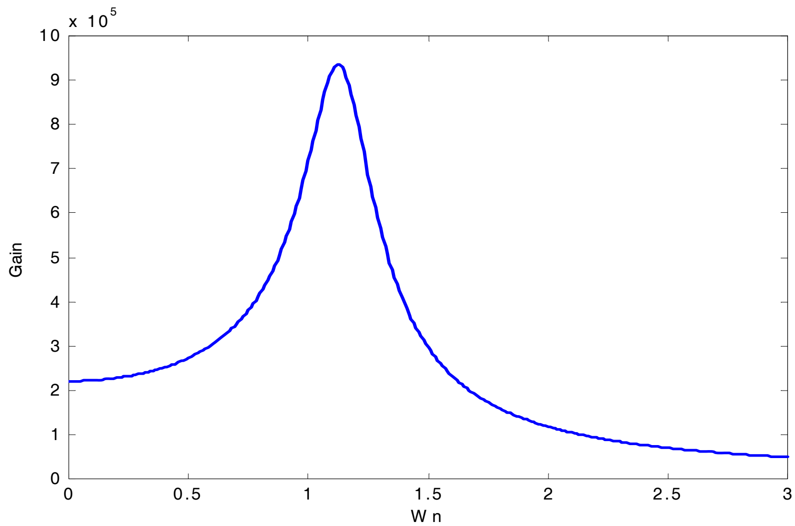
In which K_{max} is the maximum value of K for which the equation is stable. We thus observe that K_0 is a reasonable lower bound of K for stability.

For $K_0 \geq K \geq 0$, the system definitely is stable. For $K \in [0, K_0]$ we than have

$$P_1(j\omega_n) = \frac{B_0}{q} \frac{1}{\left(D_0 t_0^2 - \omega_n^2\right) + jD_1 t_0 \omega_n + \frac{B_0 K}{q} e^{-j\omega_n}} \tag{130}$$

and

$$R_1(j\omega_n) = \frac{\left(D_0 t_0^2 - \omega_n^2\right) + jD_1 t_0 \omega_n}{\left(D_0 t_0^2 - \omega_n^2\right) + jD_1 t_0 \omega_n + \frac{B_0 K}{q} e^{-j\omega_n}} \tag{131}$$



VOLTERRA ANALİZİ EŞİTLİKLERİ

$$\dot{I}_1 = A_1 \sin\,(wt+\phi_1)$$

$$\dot{I}_2 = \dot{I}_1^2 = A_1 Sin\,(wt+\phi_1)$$

$$= \frac{A_1^2}{2} - \frac{A_1^2}{2}\cos(2wt+2\phi_1)$$

$$\dot{I}_3 = C_2 \dot{I}_2 = \frac{A_1^2}{2} - \frac{A_1^2}{2}\cos\left[2wt+2\phi_1\right]$$

$$\dot{I}_4 = A_1 \left|F_3\right| \sin(wt+\phi_1+?F_3)$$

$$\left|F_3\right| = G_{33} \qquad ?F_3 = \phi_{31}$$

$$\dot{I}_4 = A_1 G_{33} \sin\left(wt+\phi_1+\phi_{31}\right)$$

$$\dot{I}_4 = A_2 \sin\left(wt+\phi_1+\phi_{31}\right)$$

$$\phi_2 = \phi_1 + \phi_{31}$$

$$A_2 = A_1 G_{33}$$

$$\dot{I}_4 = A_1 G_{33} \sin\left(wt+\phi_1\right)$$

$$\dot{I}_5 = \dot{I}_4 \dot{I}_1$$

$$= A_1^2 G_{33} \sin\left(wt+\phi_2\right)\sin\left(wt+\phi_1\right)$$

$$= \frac{A_1^2 G_{33}}{2}\cos\left(\phi_2-\phi_1\right) - \frac{A_1^2 G_{33}}{2}\cos\left[2wt+\phi_2+\phi_1\right]$$

$$= \frac{A_1^2 G_{33}}{2}\cos(\phi_{31}) - \frac{A_1^2 G_{33}}{2}\cos\left[2wt+\phi_2+\phi_1\right]$$

$$\dot{I}_6 = C_1 \dot{I}_5$$

$$= \frac{A_1^2 G_{33}}{2}\cos(\phi_{31}) - \frac{C_1 A_1^2 G_{33}}{2}\left[\cos(2wt + \phi_1)\right]$$

$$\dot{I}_7 = \dot{I}_3 + \dot{I}_6$$

$$= \frac{A_1^2 C_2}{2} - \frac{A_1^2 C_2}{2}\cos\left(2wt + 2\phi_1\right) + \frac{A_1^2 G_{33} C_1}{2}\cos\left(\phi_{31}\right)$$

$$- \frac{A_1^2 G_{33} C_1}{2}\cos\left(2wt + \phi_1 + \phi_{31}\right)$$

$$= \frac{A_1^2 C_2}{2} + \frac{A_1^2 G_{33} C_1}{2}\cos\left(\phi_{31}\right)$$

$$= \frac{A_1^2 C_2}{2}\cos\left(2wt + 2\phi_1\right) - \frac{A_1^2 G_{33} C_1}{2}\cos\left(2wt + \phi_1 + \phi_{31}\right)$$

USE MY FORMULA

$$B_1\cos\left(2wt + \psi_1\right) + B_2\cos\left(2wt + \psi_2\right)$$

$$= \gamma\cos\left(2wt + \psi_1 - \theta\right)$$

USE WHİCH

$$\gamma = \sqrt{P^2 + q^2}$$

$$\theta = \tan^{-1}\frac{q}{P}$$

$$P = B_1 + B_2\cos\left(\psi_2 - \psi_1\right)$$

$$q = B_2\sin\left(\psi_1 - \psi_2\right)$$

$$\sin(\psi) = \cos\left(\psi - \frac{\pi}{2}\right)$$

$$\sin\left(\psi + \frac{\pi}{2}\right) = \cos(\psi)$$

$$\frac{A_1^2 C_2}{2}\cos(2wt+2\phi_1)+\frac{A_1^2 G_{33} C_1}{2}\cos(2wt+\phi_1+\phi_2)$$

$$=\gamma\cos(2wt+2\phi_1-\phi)$$

$$\gamma=\sqrt{P^2+q^2}$$

$$\theta=\tan^{-1}\frac{P}{q}$$

$$P=\frac{A_1^2 C_2}{2}+\frac{A_1^2 G_{33} C_1}{2}\cos(\phi_1+\phi_2-2\phi_1)=$$

$$=\frac{A_1^2 C_2}{2}+\frac{A_1^2 G_{33} C_1}{2}\cos[\phi_2+\phi_1]$$

$$\Rightarrow P=\frac{A_1^2 C_2}{2}+\frac{A_1^2 G_{33} C_1}{2}\cos[\phi_{31}]$$

$$q=\frac{A_1^2 G_{33} C_1}{2}\sin[2\phi_1-\phi_1-\phi_2]$$

$$=\frac{A_1^2 G_{33} C_1}{2}\sin[\phi_1-\phi_2]$$

$$=\frac{A_1^2 G_{33} C_1}{2}\sin[-\phi_{31}]$$

$$q=-\frac{A_1^2 G_{33} C_1}{2}\sin[\phi_{31}]$$

$$\dot{I}_7=\frac{A_1^2 C_2}{2}+\frac{A_1^2 G_{33} C_1}{2}\cos(\phi_{31})-\cos(2wt+2\phi_1-\theta)$$

$$\gamma=\sqrt{p^2+q^2}$$

$$\theta=\tan^{-1}\frac{q}{P}$$

$$P=\frac{A_1^2 C_2}{2}+\frac{A_1^2 G_{33} C1}{2}\cos(\phi_{31})$$

$$=\frac{A_1^2}{2}[C_2+G_{33}C_1\cos(\phi_{31})]$$

$$q=\frac{A_1^2}{2}G_{33}C_1\sin(\phi_{31})$$

$$\dot{I}_7=A_1+B_7\cos(2wt+\phi_1)$$

USE WHİCH:

$$A_7 = \frac{A_1^2}{2}\left[C_2 + G_{33} C_1 \cos(\phi_{31})\right]$$

$$B_7 = -\gamma_7$$

$$\phi_7 = 2\phi_1 - \phi_1 \rightarrow \phi_7$$

$$\dot{I}_8 = A_7 F_6(S=0) + B_7 \left|F_6(S=J2w)\right| \cos(2wt + \phi_1 + ??\, F_6(S=J2w))$$

$$= A_8 + B_8 \cos(2wt + \phi_8)$$

$$A_8 = A_7 F_6(S=0)$$

$$B_8 = B_7 \left|F_6(S=J2w)\right|$$

$$\phi_8 = \phi_7 + ??\, F_6(S=J2w)$$

$$\dot{I}_{10} = A_8 F_3(S=0) + B_8 \left|F_3(S=J2w)\cos(2wt + \phi_8 + ??\, F_3(S=J2w))\right|$$

$$= A_{10} + B_{10}\cos(2wt + \phi_{10})$$

$$A_{10} = A_8 F_3(S=0)$$

$$B_{10} = B_8 \left|F_3(S=J2w)\right|$$

$$\phi_{10} = \phi_8 + ??\, F_3(S=J2w)$$

$$C_{11} = C_4 C_7$$

$$= C_4 A_7 + C_4 B_7 \cos(2wt + \phi_1)$$

$$\dot{I}_{12} = \dot{I}_{10} + \dot{I}_{11}$$

$$= A_{10} + B_{10}\cos(2wt + \phi_{10}) + C_4 A_7 + C_4 B_7 \cos(2wt + \phi_1)$$

$$= A_{10} + C_4 A_7 + B_{10}\cos(2wt + \phi_{10}) + C_4 B_7 \cos(2wt + \phi_1)$$

$$B_{10}\cos(2wt + \phi_{10}) + C_4 B_7 \cos(2wt + \phi_1)$$

$$= \gamma \cos\left[2wt + \phi_{10} - \phi\right]$$

MOW USE CALCULATE THE H_{3X} TERMS

$$H_{33} = C_7 \dot{I}_1 \dot{I}_{12}$$
$$H_{33} = C_7 A_1 \sin(wt+\phi_1)\left[A_{12} + B_{12}\cos(2wt+\phi_{12})\right]$$
$$= C_7 A_1 A_{12}\sin(wt+\phi_1) + C_7 A_1 A_{12}\sin(wt+\phi_1)\cos(2wt+\phi_{12})$$

$$\sin(wt+\phi_1)\cos(2wt+\phi_{12})$$
$$= \frac{1}{2}\sin(3wt+\phi_1+\phi_{12}) + \frac{1}{2}\sin(-wt+\phi_1-\phi_{12})$$
$$= \frac{1}{2}\sin(3wt+\phi_1+\phi_{12}) - \frac{1}{2}\sin(wt+\phi_{12}-\phi_1)$$

$$H_{33} = C_7 A_1 A_{12}\sin(wt+\phi_1) + \frac{1}{2}C_7 A_1 B_{12}\sin(3wt+\phi_1+\phi_{12})$$
$$= -\frac{1}{2}C_7 A_1 B_{12}\sin(wt+\phi_{12}-\phi_1)$$
$$= C_7 A_1 A_{12}\sin(wt+\phi_1) - \frac{1}{2}C_7 A_1 B_{12}\sin(wt+\phi_{12}-\phi_1)$$
$$+\frac{1}{2}C_7 A_1 B_{12}\sin(3wt+\phi_1+\phi_{12})$$

$$\sin[\psi] = \cos\left[\psi - \frac{\pi}{2}\right]$$

$$H_{33} = C_7 A_1 A_{12}\cos\left[wt+\phi_1-\frac{\pi}{2}\right] - \frac{1}{2}C_7 A_1 B_{12}\cos\left[wt+\phi_{12}-\phi_1-\frac{\pi}{2}\right]$$
$$+\frac{1}{2}C_7 A_1 B_{12}\cos\left[3wt+\phi_1+\phi_{12}-\frac{\pi}{2}\right]$$

NO MOW MY FORMULA OR PAGE 2

$$C_7 A_1 A_{12} \cos\left[wt + \phi_1 - \frac{\pi}{2}\right] - \frac{1}{2} C_7 A_1 B_{12} \cos\left[wt + \phi_{12} - \phi_1 - \frac{\pi}{2}\right]$$

$$= C_7 A_1 A_{12} \cos\left[wt + \phi_1 - \frac{\pi}{2}\right] + \frac{1}{2} C_7 A_1 B_{12} \cos\left[wt + \phi_{12} - \phi_1 + \frac{\pi}{2}\right]$$

$$= \gamma \cos\left[wt + \phi_1 - \frac{\pi}{2} - \theta\right]$$

$$\gamma = \sqrt{P^2 + q^2}$$

$$\theta = \tan^{-1} \frac{q}{P}$$

$$P = C_7 A_1 A_{12} + \frac{1}{2} C_7 A_1 B_{12} \cos\left[\phi_{12} - \phi_1 + \frac{\pi}{2} - \phi_1 + \frac{\pi}{2}\right]$$

$$= C_7 A_1 A_{12} + \frac{1}{2} C_7 A_1 B_{12} \cos[\phi_{12} - 2\phi_1 + \pi]$$

$$= C_7 A_1 A_{12} - \frac{1}{2} C_7 A_1 B_{12} \cos[\phi_{12} - 2\phi_1]$$

$$q = \frac{1}{2} C_7 A_1 B_{12} \sin[2\phi_1 - \phi_{12} - \pi]$$

$$= -\frac{1}{2} C_7 A_1 B_{12} \sin[2\phi_1 - \phi_{12}]$$

$$= \frac{1}{2} C_7 A_1 B_{12} \sin[\phi_{12} - 2\phi_1]$$

$$H_{33} = \gamma \cos\left[wt + \phi_1 - \frac{\pi}{2} + \theta\right] + \frac{1}{2} C_7 A_1 B_{12} \cos\left[3wt + \phi_1 + \phi_{12} - \frac{\pi}{2}\right]$$

$$H_{33} = H_{331} + H_{333}$$

$$H_{331} = \gamma \cos\left[wt + \phi_1 - \frac{\pi}{2} - \theta\right]$$

$$= m_{331} \cos[wt + \phi_{331}]$$

$$m_{331} = \gamma$$

$$\phi_{331} = \phi_1 - \frac{\pi}{2} - \theta$$

$$H_{333} = C_7 A_1 B_{12} \cos\left[3wt + \phi_1 + \phi_{12} - \frac{\pi}{2}\right]$$

$$= m_{333}\cos[3wt + \phi_{333}]$$

$$\phi_{333} = \phi_1 + \phi_{12} - \frac{\pi}{2}$$

$$m_{333} = \frac{1}{2} C_7 A_1 B_{12}$$

$$H_{31} = C_5 \dot{I}_1 \dot{I}_8$$

$$= C_5 A_1 A_8 \sin(wt + \phi_1)[A_8 + B_8 \cos(2wt + \phi_8)]$$

$$= C_5 A_1 A_8 \sin[(wt + \phi_1)]$$

$$+ C_5 A_1 B_8 \sin(wt + \phi_1)\cos(2wt + \phi_8)$$

$$\sin(wt + \phi_1)\cos(2wt + \phi_8) = \frac{1}{2}\sin(3wt + \phi_1 + \phi_8) - \frac{1}{2}\sin(wt + \phi_8 - \phi_1)$$

$$H_{31} = C_5 A_1 A_8 \sin(wt + \phi_1) - \frac{1}{2} C_5 A_1 B_8 \sin(wt + \phi_8 - \phi_1)$$

$$+ \frac{1}{2} C_5 A_1 B_8 \sin(3wt + \phi_1 + \phi_8)$$

$$\sin(\psi) = \cos\left(\psi - \frac{\pi}{2}\right)$$

$$H_{31} = C_5 A_1 A_8 \cos(wt + \phi_1 - \frac{\pi}{2}) - \frac{1}{2} C_5 A_1 B_8 \cos(wt + \phi_8 - \phi_1 - \frac{\pi}{2})$$

$$+ \frac{1}{2} C_5 A_1 B_8 \cos(3wt + \phi_1 + \phi_8 - \frac{\pi}{2})$$

MOW USİNG MY FORMULA ON PAGE 2

$$C_5 A_1 A_8 \cos\left[wt + \phi_1 - \frac{\pi}{2}\right] - \frac{1}{2} C_5 A_1 B_8 \cos\left[wt + \phi_8 - \phi_1 - \frac{\pi}{2}\right]$$

$$= C_5 A_1 A_8 \cos\left[wt + \phi_1 - \frac{\pi}{2}\right] + \frac{1}{2} C_5 A_1 B_8 \cos\left[wt + \phi_8 - \phi_1 + \frac{\pi}{2}\right]$$

$$= \gamma \cos\left[wt + \phi_1 - \frac{\pi}{2} - \theta\right]$$

$$\gamma = \sqrt{P^2 + q^2}$$

$$\theta_{m6} = \tan^{-1}\frac{q}{P}$$

$$P = C_5 A_1 A_8 + \frac{1}{2} C_5 A_1 B_8 \cos\left(\phi_8 - \phi_1 + \frac{\pi}{2} - \phi_1 + \frac{\pi}{2}\right)$$

$$= C_5 A_1 A_8 + \frac{1}{2} C_5 A_1 B_8 \cos(\phi_8 - 2\phi_1 + \pi)$$

$$= C_5 A_1 A_8 - \frac{1}{2} C_5 A_1 B_8 \cos(\phi_8 - 2\phi_1)$$

$$q = \frac{1}{2} C_5 A_1 A_8 \sin(2\phi_1 - \phi_8 - \pi)$$

$$= \frac{1}{2} C_5 A_1 B_8 \sin(\phi_8 - 2\phi_1)$$

$$H_{31} = \gamma \cos\left[wt + \phi_1 - \frac{\pi}{2} - \theta\right] + \frac{1}{2} C_5 A_1 B_8 \cos\left(3wt + \phi_1 + \phi_8 - \frac{\pi}{2}\right)$$

$$= H_{311} + H_{313}$$

$$H_{311} = m_{61} \cos[wt + \phi_{61}]$$

$$m_{61} = \gamma$$

$$\phi_{61} = \phi_1 - \frac{\pi}{2} - \theta$$

$$H_{313} = m_{63} \cos[3wt + \phi_{63}]$$

$$m_{63} = \frac{1}{2} C_5 A_1 B_8$$

$$\phi_{63} = \phi_1 + \phi_8 - \frac{\pi}{2}$$

$$H_{32} = C_6 \dot{I}_4 \dot{I}_8$$

$$= C_6 A_1 G_{33} \sin(wt + \phi_2)[A_8 + B_8 \cos(2wt + \phi_8)]$$

$$C_5 \to C_6 G_{33}$$

$$H_{32} = \gamma \cos\left[wt + \phi_1 - \frac{\pi}{2} - \theta\right] + \frac{1}{2} C_6 G_{33} A_1 B_8 \cos\left[3wt + \phi_1 + \phi_8 - \frac{\pi}{2}\right]$$

$$\gamma = \sqrt{P^2 + q^2}$$

$$\theta = \tan^{-1}\frac{q}{P}$$

$$P = C_6 G_{33} A_1 A_8 - \frac{1}{2} C_6 G_{33} A_1 B_8 \cos(\phi_8 - 2\phi_1)$$

$$q = \frac{1}{2} C_6 G_{33} A_1 B_8 - \sin(\phi_8 - 2\phi_1)$$

$$H_{32} = H_{321} + H_{323}$$

$$H_{321} = m_{71} \cos[wt + \phi_{71}]$$

$$m_{71} = \gamma$$

$$\phi_{71} = \phi_1 - \frac{\pi}{2} - \theta$$

$$H_{323} = m_{73} \cos[3wt + \phi_{73}]$$

$$m_{73} = \frac{1}{2} C_6 G_{33} A_1 B_8$$

$$\phi_{73} = \phi_1 + \phi_8 - \frac{\pi}{2}$$

$$H_{34} = C_8 \dot{I}_2 \dot{I}_4$$

$$\dot{I}_2 = \frac{A_1^2}{2} - \frac{A_1^2}{2} \cos(2wt + 2\phi_1)$$

$$\dot{I}_4 = A_1 G_{33} \sin(wt + \phi_2)$$

$$H_{34} = \frac{A_1^2}{2} [1 - \cos(2wt + 2\phi_1)] A_1 G_{33} \sin(wt + \phi_2)$$

$$= \frac{1}{2} A_1^3 G_{33} [1 - \cos(2wt + 2\phi_1)][\sin(wt + \phi_2)]$$

$$C_5 A_1 \to \frac{1}{2} A_1^3 G_{33}$$

$$\phi_1 \to \phi_2$$

$$\phi_8 \to 2\phi_1$$

$$A_8 = 1$$

$$B_8 = -1$$

$$H_{34} = \gamma \cos\left(wt + \phi_2 - \frac{\pi}{2} - \theta\right) + \frac{1}{2}\frac{1}{2} A_1^3 G_{33} (-1) \cos\left[3wt + \phi_2 + 2\phi_1 - \frac{\pi}{2}\right]$$

$$= \gamma \cos\left(wt + \phi_2 - \frac{\pi}{2} - \theta\right) - \frac{1}{4} A_1^3 G_{33} \cos\left[3wt + \phi_2 + 2\phi_1 - \frac{\pi}{2}\right]$$

$$\gamma \cos\left(wt + \phi_2 - \frac{\pi}{2} - \theta \right) + \frac{1}{4} A_1^3 G_{33} \cos\left[3wt + \phi_1 + 2\phi_1 + \frac{\pi}{2} \right]$$

$$\gamma = \sqrt{P^2 + q^2}$$

$$\theta = \tan^{-1} \frac{q}{P}$$

$$P = \frac{1}{2} A_1^3 G_{33} + \frac{1}{2}\frac{1}{2} A_1^3 G_{33} \cos[2\phi_1 - 2\phi_2]$$

$$= \frac{1}{2} A_1^3 G_{33} + \frac{1}{4} A_1^3 G_{33} \cos[2(\phi_1 - \phi_2)]$$

$$q = \frac{1}{2}\frac{1}{2} A_1^3 G_{33} (-1) \sin[2\phi_1 - 2\phi_2]$$

$$= -\frac{1}{4} A_1^3 G_{33} \sin[2(\phi_1 - \phi_2)]$$

$$= +\frac{1}{4} A_1^3 G_{33} \sin[2(\phi_2 - \phi_1)]$$

$$H_{34} = H_{341} + H_{343}$$

$$H_{341} = m_{81} \cos[wt + \phi_{81}]$$

$$m_{81} = \gamma$$

$$\phi_{81} = \phi_2 - \frac{\pi}{2} - \theta$$

$$m_{83} = \frac{1}{4} A_1^3 G_{33}$$

$$\phi_{83} = \phi_2 + 2\phi_1 + \frac{\pi}{2}$$

WE ARE MOW AT

$$H_{33} = H_{331} + H_{333}$$

$$H_{31} = H_{311} + H_{313}$$

$$H_{32} = H_{321} + H_{323}$$

$$H_{34} = H_{341} + H_{343}$$

$$({}_{331} + H_{311}) = m_{331} \cos(wt + \phi_{331}) + m_{61} \cos(wt + \phi_{m61})$$

$\gamma_{a1} = \cos\left(wt + \phi_{331} - \theta_{a!}\right)$

$\gamma_{200} \to \gamma_{a1} = \sqrt{P_{a1}^2 + q_{a1}^2}$

$\theta_{a1} = \tan^{-1}\dfrac{q_a}{p_a} \leftarrow \theta_{200}$

$P_{200} \to P_{a1} = m_{331} + m_{61}\cos(\theta_{61} - \theta_{331})$

$q_{200} \to q_{a1} = m_{61}\sin(\theta_{331} - \theta_{61})$

$H_{331} + H_{311} + H_{321} = \gamma_{a1}\cos(wt + \phi_{331} - \theta_{a1}) + m_{71}\cos(wt + \phi_{71})$

$= \gamma_{b1}\cos(wt + \phi_{331} - \theta_{a1} - \theta_{b1})$

$\gamma_{201} \to \gamma_{b1} = \sqrt{P_{b1}^2 + q_{a1}^2}$

$\theta_{b1} = \tan^{-1}\dfrac{q_{b1}}{p_{b1}} \leftarrow \theta_{201}$

$P_{201} \to P_{b1} = \gamma_{a1} + m_{61}\cos(\theta_{71} - \theta_{331} + \theta_{a1})$

$q_{201} \to q_{b1} = m_{71}\sin(\theta_{331} - \theta_{a1} - \theta_{71})$

$S_{ma1} = (H_{331} + H_{321} + H_{321}) + H_{341}$

$S_{ma1} = \gamma_{b1}\cos(wt + \theta_{331} - \theta_{a1} - \theta_{b1}) + m_{81}\cos(wt + \theta_{81})$

$S_{ma1} = \gamma_{b1}\cos(wt + \theta_{331} - \theta_{a1} - \theta_{b1} - \theta_{c1})$

$\gamma_{c1} = \sqrt{P_{c1}^2 + q_{c1}^2}$

$\theta_{c1} = \tan^{-1}\dfrac{q_{c1}}{p_{c1}}$

$P_{c1} = \gamma_{b1} + m_{81}\cos(\phi_{81} - \phi_{331} + \theta_{a1} + \theta_{b1})$

$q_{c!} = m_{81}\sin(\phi_{331} - \theta_{a1} - \theta_{b1} - \phi_{81})$

$S_{m23} = H_{333} + H_{313} + H_{323} + H_{343}$

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$H_{333} + H_{313} = m_{333}\cos(3wt + \phi_{333}) + m_{63}\cos(3wt + \phi_{63})$

$= \gamma_{a3}\cos(3wt + \phi_{333} - \phi_{a3})$

$\gamma_{230} \to \gamma_{a3} = \sqrt{P_{a3}^2 + q_{a3}^2}$

$\theta_{a3} = \tan^{-1}\dfrac{q_{a3}}{p_{a3}}$

$$P_{230} \to P_{a3} = m_{333} + m_{63}\cos(\phi_{63} - \phi_{333})$$
$$q_{230} \to q_{a3} = m_{63}\sin(\phi_{333} - \phi_{63})$$
$$(H_{333} + H_{313}) + H_{323} = \gamma_{a3}\cos(3wt + \phi_{333} - \phi_{a3}) + m_{63}\cos(3wt + \phi_{73})$$
$$= \gamma_{b3}\cos(3wt + \phi_{333} - \phi_{a3} - \theta_{b3})$$
$$\gamma_{231} \to \gamma_{b3} = \sqrt{P_{b3}^2 + q_{b3}^2}$$
$$\theta_{b3} = \tan^{-1}\frac{q_{b3}^2}{P_{b3}^2} \leftarrow \theta_{231}$$
$$P_{231} \to P_{b3} = \gamma_{a3} + m_{73}\cos(\phi_{73} - \phi_{333} + \theta_{a3})$$
$$q_{231} \to q_{b3} = m_{73}\sin(\phi_{333} - \phi_{a3} - \phi_{73})$$

$$S_{ma3} = \left(H_{333} + H_{313} + H_{323}\right) + H_{343}$$
$$= \gamma_{b3}\cos(3wt + \phi_{333} - \theta_{a3} - \theta_{b3}) + m_{83}\cos(3wt + \phi_{83})$$
$$= \gamma_{c3}\cos(3wt + \phi_{333} - \theta_{a3} - \theta_{b3} - \theta_{c3})$$
$$\gamma_{232} = \gamma_{c3} = \sqrt{P_{c3}^2 + q_{c3}^2}$$
$$\theta_{c3} = \tan^{-1}\frac{q_{c3}^2}{P_{c3}^2} \leftarrow \theta_{232}$$
$$P_{232} \to P_{c3} = \gamma_{b3} + m_{83}\cos(\phi_{83} - \phi_{333} + \theta_{a3} + \theta_{b3})$$
$$q_{232} \to q_{c3} = m_{83}\sin(\phi_{333} - \theta_{a3} - \theta_{b3} - \phi_{83})$$

$$S_{m20} = S_{m21} + S_{m23}$$
$$S_{m21} = \gamma_{c1}\cos(wt + \phi_{331} - \theta_{a1} - \theta_{b1} - \theta_{c1})$$
$$S_{m23} = \gamma_{c3}\cos(3wt + \phi_{331} - \theta_{a3} - \theta_{b3} - \theta_{c3})$$
$$F10 = F6.R1$$
$$PH_{31} = PH_{311} + PH_{313}$$
$$PH_{311} = \left|F10(S = Jw)\right|\gamma_{c1}\cos(wt + \phi_{331} - \theta_{a1} - \theta_{b1} - \theta_{c1}) + \kappa F10(S = Jw)$$
$$PH_{313} = \left|F10(S = J3w)\right|\gamma_{c3}\cos(3wt + \phi_{331} - \theta_{a3} - \theta_{b3} - \theta_{c3}) + \kappa F10(S = J3w)$$
$$\dot{I}_8 = A_8 + B_8\cos(2wt + \phi_8)$$
$$\dot{I}_{14} = -A_8Kq_0 + B_8Kq_0\cos\left[2wt + \phi_8 + \pi - 2w\right]$$

$$\dot{I}_{15} = \dot{I}_{14} + \dot{I}$$
$$= \dot{I}_{14} + \mu \sin(wt)$$
$$= -A_8 K q_0 + \mu Sin(wt) + B_8 K q_0 \cos[2wt + \phi_8 + \pi - 2w]$$
$$\dot{I}_{16} = -A_8 K q_0 Z_1(S=0) + \mu |Z_1(S=Jw)| \sin[wt + \kappa Z_1(S=jw)]$$
$$+ A_8 K q_0 \cos[2wt + \phi_8 + \pi - 2w + \kappa Z_1(S=j2w)]$$
$$= A_{16} + C_{16} \sin[wt + \Phi_{11}] + B_{16} \cos[2wt + \phi_{162}]$$
$$A_{16} = -A_8 K q_0 Z_1(S=0)$$
$$C_{16} = \mu |Z_1(S=Jw)|$$
$$\phi_{161} \rightarrow \phi_{11} = \kappa Z_1(S=Jw) \rightarrow \phi_{11}$$
$$B_{16} = B_8 K q_0 |Z_1(S=j2w)|$$
$$\phi_{162} = \phi_8 + \pi - 2w + \kappa Z_1(S=J2w)$$
$$\dot{I}_{12} = \dot{I}_{16}^2 = [A_{16} + C_{16}\sin(wt+\phi_{11}) + B_{16}\cos(2wt+\phi_{162})]$$
$$= 2A_{16}C_{16}\sin(wt+\phi_{11}) + 2C_{16}B_{16}\sin(wt+\phi_{11})\cos(2wt+\phi_{162})$$
$$= 2A_{16}C_{16}\sin(wt+\phi_{11}) + C_{16}B_{16}\sin(3wt+\phi_{11}+\phi_{162})$$
$$+ C_{16}B_{16}\sin(-wt+\phi_{11}-\phi_{162})$$
$$= 2A_{16}C_{16}\sin(wt+\phi_{11}) - C_{16}B_{16}\sin(wt-\phi_{11}+\phi_{162}) + C_{16}B_{16}\sin(3wt+\phi_{11}+\phi_{162})$$

$$2A_{16}C_{16}\cos(wt+\phi_{11}-\frac{\pi}{2}) + C_{16}B_{16}\cos(wt-\phi_{11}+\phi_{162}+\frac{\pi}{2})$$
$$= \gamma_{17}\cos(wt+\phi_{11}-\frac{\pi}{2}-\phi_{17})$$
$$\gamma_{17} = \sqrt{P_{17}^2 + q_{17}^2}$$
$$\theta_{17} = \tan^{-1}\frac{q_{17}}{P_{17}}$$
$$P_{17} = 2A_{16}C_{16} + C_{16}B_{16}Cos(\phi_{162} - \phi_{11} + \frac{\pi}{2} - \phi_{11} + \frac{\pi}{2})$$
$$= 2A_{16}C_{16} - C_{16}B_{16}Cos(\phi_{162} - 2\phi_{11})$$
$$q_{17} = C_{16}B_{16}Sin(\phi_{11} - \frac{\pi}{2} - \phi_{162} + \phi_{11} - \frac{\pi}{2})$$
$$= -C_{16}B_{16}Sin(2\phi_{11} - \phi_{162})$$

$$\dot{I}_{17} = \gamma_{17}\cos(wt + \phi_{11} - \frac{\pi}{2} - \theta_{17}) + C_{16}B_{16}\sin(3wt + \phi_{11} + \phi_{162})$$
$$= \gamma_{17}\cos(wt + \psi_{171}) + C_{16}B_{16}\sin(3wt + \psi_{173})$$
$$\psi_{171} = \phi_{11} - \frac{\pi}{2} - \theta_{17}$$
$$\psi_{173} = \phi_{11} + \phi_{162}$$

$$\dot{I}_{18} = C_2\dot{I}_{17}$$
$$= C_2\gamma_{17}\cos(wt + \psi_{171}) + C_2C_{16}B_{16}\sin(3[wt + \psi_{173}])$$
$$\dot{I}_{180} = A_{18}.\cos(wt + \psi_{181}) + B_{18}\sin(3wt + \psi_{183})$$
$$A_{18} = C_2\gamma_{17}$$
$$B_{18} = C_2C_{16}B_{16}$$
$$\dot{I}_{19} = A_{16}F_3(S=0) + C_{16}\left|F_3(S = jw)\right|\sin\left[wt + \phi_{11} + \kappa F_3(S = jw)\right]$$
$$+ B_{16}\left|F_3(S = j2w)\right|\cos\left[2wt + \phi_{162} + \kappa F_3(S = j2w)\right]$$
$$\dot{I}_{19} = A_{19} + C_{19}\sin[wt + \psi_{191}] + B_{19}\cos[2wt + \psi_{192}]$$
$$A_{19} = A_{16}F_3(S=0) \rightarrow F_{30}$$
$$C_{19} = C_{16}\left|F_3(S = jw)\right| \rightarrow F_{31}$$
$$B_{19} = B_{16}\left|F_3(S = j2w)\right| \rightarrow F_{32}$$
$$\psi_{191} = \phi_{11} + \kappa F_3(S = jw) \rightarrow F_{31} \rightarrow \phi_{31}$$
$$\psi_{192} = \phi_{162} + \kappa F_3(S = j2w)F_{32} \rightarrow \phi_{32}$$
$$\dot{I}_{20} = \dot{I}_{16}.\dot{I}_{19}$$
$$\dot{I}_{20} = \left[A_{19} + C_{19}\sin[wt + \psi_{191}] + B_{19}\cos[2wt + \psi_{192}]\right].$$
$$\left[A_{16} + C_{16}\sin[wt + \phi_{11}] + B_{16}\cos[2wt + \phi_{162}]\right]$$

AGAİN WE ARE ONLY İNTERESTED İN THE .... HARMONİC TERMS. THEY ARE

$$= A_{19}C_{16}\sin(wt + \phi_{11}) + A_{16}C_{19}\sin(wt + \psi_{191}) + C_{19}B_{16}\sin(wt + \psi_{191}).$$
$$\cos(2wt + \phi_{162}) + C_{16}B_{19}\sin(wt + \phi_{11})\cos(2wt + \psi_{192})$$
$$= A_{19}C_{16}\sin(wt + \phi_{11}) \mp A_{16}C_{19}\sin(wt + \psi_{191}) + \frac{1}{2}C_{19}B_{16}\sin(3wt + \psi_{191} + \phi_{162})$$

$$+\frac{1}{2}C_{19}B_{16}\sin(-wt+\psi_{191}-\phi_{162})+\frac{1}{2}C_{16}B_{19}\sin(3wt+\phi_{11}+\psi_{192})$$
$$+\frac{1}{2}C_{16}B_{19}\sin(-wt+\phi_{11}-\psi_{192})$$

$$\dot{I}_{20}=A_{19}C_{16}\sin(wt+\phi_{11})+A_{16}C_{19}\sin(wt+\psi_{191})$$
$$-\frac{1}{2}C_{19}B_{16}\sin(3wt+\phi_{162}-\psi_{191})-\frac{1}{2}C_{16}B_{19}\sin(wt-\phi_{11}+\psi_{192})$$
$$+\frac{1}{2}C_{19}B_{16}\sin(3wt+\phi_{162}+\psi_{191})+\frac{1}{2}C_{16}B_{19}\sin(3wt+\phi_{11}+\psi_{192})$$
$$A_{19}C_{16}\cos(wt+\phi_{11}-\frac{\pi}{2})+A_{16}C_{19}\cos(wt+\psi_{191}-\frac{\pi}{2})$$
$$=\gamma_{20a}\cos(wt+\phi_{11}-\frac{\pi}{2}-\theta_{20a})$$
$$\gamma_{300}=\gamma_{20a}=\sqrt{P_{20a}^2+q_{20a}^2}$$
$$\theta_{20a}=\tan^{-1}\frac{q_{20a}}{P_{20a}}$$
$$P_{300}\rightarrow P_{20a}=A_{19}C_{16}+A_{16}C_{19}\cos(\psi_{191}-\frac{\pi}{2}-\phi_{11}+\frac{\pi}{2})$$
$$P_{300}\rightarrow P_{20a}=A_{19}C_{16}+A_{16}C_{19}\cos(\psi_{191}-\phi_{11})$$
$$q_{300}=q_{20a}=A_{16}C_{19}\sin\left[\phi_{11}-\frac{\pi}{2}-\psi_{191}+\frac{\pi}{2}\right]$$
$$q_{20a}=A_{16}C_{19}\sin[\phi_{11}-\psi_{191}]$$
$$=\gamma_{20a}\cos\left[wt+\phi_{11}-\frac{\pi}{2}-\theta_{20a}\right]+\frac{1}{2}C_{19}B_{16}\cos\left[wt-\psi_{191}+\phi_{162}+\frac{\pi}{2}\right]$$
$$\gamma_{30b}=\gamma_{20b}\cos\left[wt+\phi_{11}-\frac{\pi}{2}-\theta_{20a}-\theta_{20b}\right]$$
$$\gamma_{20b}=\sqrt{P_{20b}^2+q_{20b}^2}$$
$$\theta_{20b}=\tan^{-1}\frac{q_{20b}}{P_{20b}}$$
$$P_{30b}=P_{20b}=\gamma_{20a}+\frac{1}{2}C_{19}B_{16}\cos\left[-\psi_{191}+\phi_{162}+\frac{\pi}{2}-\phi_{11}+\frac{\pi}{2}+\theta_{20a}\right]$$

$$= \gamma_{20a} - \frac{1}{2} C_{19} B_{16} \cos[\phi_{162} - \psi_{191} - \phi_{11} + \theta_{20a}]$$

$$q_{30b} = q_{20b} = \frac{1}{2} C_{19} B_{16} \sin\left[\phi_{11} - \frac{\pi}{2} - \theta_{20a} + \psi_{191} - \phi_{162} - \frac{\pi}{2}\right]$$

$$= +\frac{1}{2} C_{19} B_{16} \sin[\phi_{162} - \psi_{191} - \phi_{11} + \theta_{20a}]$$

$$\gamma_{301} = \gamma_{20b} \cos\left[wt + \phi_{11} - \frac{\pi}{2} - \theta_{20a} - \theta_{20b}\right] - \frac{1}{2} C_{16} B_{19} \sin[wt - \phi_{11} - \psi_{192}]$$

$$= \gamma_{20b} \cos\left[wt + \phi_{11} - \frac{\pi}{2} - \theta_{20a} - \theta_{20b}\right] + \frac{1}{2} C_{16} B_{19} \sin\left[wt - \phi_{11} + \phi_{192} + \frac{\pi}{2}\right]$$

$$\gamma_{302} = \gamma_{201} \cos\left[wt + \phi_{11} - \frac{\pi}{2} - \theta_{20a} - \theta_{20b} - \theta_{20C}\right]$$

$$\gamma_{302} \rightarrow \gamma_{201} = \sqrt{P_{20C}^2 + q_{20C}^2}$$

$$\theta_{20c} = \tan^{-1} \frac{q_{20C}}{P_{20C}}$$

$$P_{20c} = \gamma_{20b} + \frac{1}{2} C_{16} B_{19} \cos\left[-\phi_{11} + \psi_{192} + \frac{\pi}{2} - \phi_{11} + \frac{\pi}{2} + \theta_{20a} + \theta_{20b}\right]$$

$$= \gamma_{20b} - \frac{1}{2} C_{16} B_{19} \cos[+\psi_{192} - 2\phi_{11} + \theta_{20a} + \theta_{20b}]$$

$$q_{20c} = \frac{1}{2} C_{16} B_{19} \sin\left[\phi_{11} - \frac{\pi}{2} - \theta_{20a} - \theta_{20b} + \phi_{11} - \psi_{192} - \frac{\pi}{2}\right]$$

$$= -\frac{1}{2} C_{16} B_{19} \sin[2\phi_{11} - \psi_{192} - \theta_{20a} - \theta_{20b}]$$

$$= +\frac{1}{2} C_{16} B_{19} \sin[\psi_{192} - 2\phi_{11} + \theta_{20a} + \theta_{20b}]$$

$$= \frac{1}{2} C_{19} B_{16} \sin[3wt + \psi_{192} + \phi_{162}] + \frac{1}{2} C_{16} B_{19} \sin[3wt + \psi_{192} + \phi_{11}]$$

$$= \frac{1}{2}\left[C_{19} B_{16} \cos\left[3wt + \psi_{192} + \phi_{162} - \frac{\pi}{2}\right] + C_{16} B_{19} \cos\left[3wt + \psi_{192} + \phi_{11} - \frac{\pi}{2}\right]\right]$$

$$= \gamma_{203} \cos\left[3wt + \psi_{192} + \phi_{162} - \frac{\pi}{2} - \theta_{203}\right]$$

$$\gamma_{203} = \frac{1}{2}\sqrt{P_{203}^2 + q_{203}^2}$$

$$P_{203} = C_{19}B_{16} + C_{16}B_{19}\cos\left[\phi_{11} + \psi_{192} - \frac{\pi}{2} - \psi_{191} - \phi_{162} + \frac{\pi}{2}\right]$$
$$= C_{19}B_{16} + C_{16}B_{19}\cos[\phi_{11} + \psi_{192} - \psi_{191} - \phi_{162}]$$
$$q_{203} = C_{16}B_{19}\sin\left[\psi_{191} + \phi_{162} - \frac{\pi}{2} - \phi_{11} - \psi_{192} + \frac{\pi}{2}\right]$$
$$= C_{16}B_{19}\sin[\psi_{191} + \phi_{162} - \phi_{11} - \psi_{192}]$$

$$\dot{I}_{20} = \gamma_{201}\cos\left[wt + \phi_{11} - \frac{\pi}{2} - \theta_{20a} - \theta_{20b} - \theta_{20c}\right]$$
$$+ \gamma_{201}\cos\left[3wt + \psi_{191} + \phi_{162} - \frac{\pi}{2} - \theta_{203}\right]$$
$$\dot{I}_{21} = C_1\dot{I}_{20}$$
$$= C_1\gamma_{201}\cos\left[wt + \phi_{11} - \frac{\pi}{2} - \theta_{20a} - \theta_{20b} - \theta_{20c}\right]$$
$$+ C_1\gamma_{203}\cos\left[3wt + \psi_{191} + \phi_{162} - \frac{\pi}{2} - \theta_{203}\right]$$
$$\dot{I}_{22} = \dot{I}_{18} + \dot{I}_{21}$$
$$= C_2\gamma_{17}\cos[wt + \psi_{191}] + C_2C_{16}B_{16}\sin[3wt + \psi_{173}] + C_1\gamma_{201}\cos[wt + \psi_{211}]$$
$$+ C_1\gamma_{203}\cos[3wt + \psi_{213}]$$
$$\psi_{211} = \left[\phi_{11} - \frac{\pi}{2} - \theta_{20a} - \theta_{20b} - \theta_{20c}\right]$$
$$\psi_{213} = \left[\psi_{191} + \phi_{162} - \frac{\pi}{2} - \theta_{203}\right]$$

SO FIRST HARMONIC .....??

$$= C_2\gamma_{17}\cos[wt + \psi_{171}] + C_2\gamma_{201}\cos[wt + \psi_{211}]$$
$$= C_{221}\cos[wt + \psi_{171} - \theta_{221}]$$
$$\gamma_{221} = \sqrt{P_{221}^2 + q_{221}^2}$$
$$P_{221} = C_2\gamma_{17} + C_1\gamma_{201}\cos[\psi_{221} - \psi_{171}]$$
$$q_{221} = C_1\gamma_{201}\sin[\psi_{171} - \psi_{211}]$$

AND THE 3 ND HARMANİC ...??

$$= C_2 C_{16} B_{16} \sin[3wt + \psi_{173}] + C_1 \gamma_{203} \cos[3wt + \psi_{213}]$$
$$= C_2 C_{16} B_{16} \cos\left[3wt + \psi_{173} - \frac{\pi}{2}\right] + C_1 \gamma_{203} \cos[3wt + \psi_{213}]$$
$$= \gamma_{223} \cos\left[3wt + \psi_{173} - \frac{\pi}{2} - \theta_{223}\right]$$
$$\gamma_{223} = \sqrt{P_{223}^2 + q_{223}^2}$$
$$\theta_{223} = \tan^{-1} \frac{q_{223}}{P_{223}}$$
$$P_{223} = C_2 C_{16} B_{16} + C_1 \gamma_{203} \cos\left[\psi_{213} - \psi_{173} + \frac{\pi}{2}\right]$$
$$q_{223} = C_1 \gamma_{203} \sin\left[\psi_{173} - \frac{\pi}{2} - \psi_{213}\right]$$
$$\dot{I}_{22} = \gamma_{221} \cos[wt + \psi_{171} - \theta_{221}] + \gamma_{223} \cos\left[3wt + \psi_{173} - \frac{\pi}{2} - \theta_{223}\right]$$
$$PH_{32} = \gamma_{221} |F_{10}(S = jw)| \cos[wt + \psi_{171} - \theta_{221} + \kappa F_{10}(S = jw)]$$
$$+ \gamma_{223} |F_{10}(S = j3w)| \cos\left[3wt + \psi_{173} - \frac{\pi}{2} - \theta_{223} + \kappa F_{10}(S = j3w)\right]$$
$$= PH_{321} + PH_{321}$$
$$PH_{321} = \gamma_{221} |F_{10}(S = jw)| \cos[wt + \psi_{171} - \theta_{221} + \kappa F_{10}(S = jw)]$$
$$PH_{323} = \gamma_{223} |F_{10}(S = j3w)| \cos\left[3wt + \psi_{173} - \frac{\pi}{2} - \theta_{223} + \kappa F_{10}(S = j3w)\right]$$
$$P_3 = PH_{31} + PH_{32} = PH_{311} + PH_{313} + PH_{321} + PH_{323}$$
$$= P_{P31} + P_{P32}$$
$$P_{P31} = PH_{311} + PH_{321} = |F_{10}(S = j3w)| \gamma_{C1} \cos$$
$$[wt + \phi_{331} - \theta_{a1} - \theta_{b1} - \theta_{c1} + \kappa F_{10}(S = jw)]$$
$$+ \gamma_{221} |F_{10}(S = j3w)| \cos[wt + \psi_{171} - \theta_{221} + \kappa F_{10}(S = jw)]$$
$$= Z_{31} Cos[wt + \phi_{331} - \theta_{a1} - \theta_{b1} - \theta_{c1} + \kappa F_{10}(S = jw)]$$

$$Z_{31} = \left|F_{10}(S = jw)\right|\sqrt{P_{31}^2 + q_{31}^2}$$

$$\theta_{P31} = \tan^{-1}\frac{q_{31}}{P_{31}}$$

$$P_{31} = \gamma_{C1} + \gamma_{221}\cos\left[\psi_{171} - \theta_{221} - \phi_{331} + \theta_{a1} + \theta_{b1} + \theta_{c1}\right]$$

$$q_{31} = \gamma_{221}\sin\left[\theta_{221} - \psi_{171} - \phi_{331} - \theta_{a1} - \theta_{b1} - \theta_{c1}\right]$$

$$P_{P33} = PH_{313} + PH_{323}$$

$$= \gamma_{C3}\left|F_{10}(S = jw)\right|\cos\left[3wt + \phi_{333} - \theta_{a1} - \theta_{b1} - \theta_{c1} + \kappa F_{10}(S = j3w)\right]$$

$$+ \gamma_{223}\left|F_{10}(S = j3w)\right|\cos\left[3wt + \psi_{123} - \frac{\pi}{2} - \theta_{233} + \kappa F_{10}(S = j3w)\right]$$

$$= Z_{33}\cos\left[3wt + \phi_{333} - \theta_{a3} - \theta_{b3} - \theta_{c3} + \kappa F_{10}(S = j3w) - \theta_{P3}\right]$$

$$Z_{33} = \left|F_{10}(S = j3w)\right|\sqrt{P_{33}^2 + q_{33}^2}$$

$$\theta_{P33} = \tan^{-1}\frac{q_3}{P_{33}}$$

$$P_{33} = \gamma_{C3} + \gamma_{223}\cos\left[\psi_{123} - \frac{\pi}{2} - \theta_{233} - \phi_{333} - \theta_{a3} + \theta_{b3} + \theta_{C3}\right]$$

$$= \gamma_{C3} + \gamma_{223}\cos\left[\psi_{123} - \theta_{233} - \phi_{333} + \theta_{a3} + \theta_{b3} + \theta_{C3}\right]$$

$$q_{33} = \gamma_{223}\sin\left[\phi_{333} - \theta_{a3} - \theta_{b3} - \theta_{C3} - \psi_{173} + \frac{\pi}{2} + \theta_{223}\right]$$

$$= \gamma_{223}\cos\left[\phi_{333} - \theta_{a3} - \theta_{b3} - \theta_{C3} - \psi_{173} + \theta_{223}\right]$$

$$P_3 = P_{P31} + P_{P33}$$

$$P_{P31} = Z_{31}\cos\left[wt + \phi_{331} - \theta_{a1} - \theta_{b1} - \theta_{c1} + \kappa F_{10}(S = jw) - \theta_{P3}\right]$$

$$P_{P33} = Z_{33}\cos\left[3wt + \phi_{333} - \theta_{a3} - \theta_{b3} - \theta_{c3} + \kappa F_{10}(S = j3w) - \theta_{P3}\right]$$