

FUZZY PARAMETERIZED FUZZY SOFT MATRICES AND THEIR APPLICATION IN DECISION-MAKING

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ABSTRACT. In this study, we define the concept of fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) and present some of their basic properties. By using *fpfs*-matrices, we then suggest a new algorithm, i.e. Prevalence Effect Method (PEM), and apply this method to a performance-based value assignment, so that we can order noise removal filters regarding performance. The results show that PEM has a potential for several areas, such as machine learning and image processing. Finally, we discuss *fpfs*-matrices and PEM for further research.

Keywords: Fuzzy sets, soft sets, soft matrices, *fpfs*-matrices, prevalence effect method

AMS Subject Classification: 03E72, 03E75, 15B15

1. INTRODUCTION

The concept of soft sets, the graphic of a function from a parameter set to the power set of a universal set, was defined by Molodtsov [1] in 1999 to deal with many problems containing uncertainties and so far a broad range of theoretical and applied studies from algebra to decision-making have been conducted on this concept [2–43].

In 2010, Çağman et al. [6] introduced the fuzzy parameterized fuzzy soft sets (*fpfs*-sets) because back then a more general form was needed for mathematical modelling of some problems in the event of parameters or objects with uncertainties - today such a need still exists. However, in the case that a large body of data is processed, computer mathematics should be employed. To deal with this problem, in Section 2, we propose the concept of fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) and investigate some of their basic properties. This concept is first mentioned in the first author's PhD dissertation. In Section 3, by using *fpfs*-matrices we describe a fast and simple algorithm named Prevalence Effect Method (PEM). In Section 4, we apply this method to a performance-based value assignment problem. Finally, we discuss *fpfs*-matrices and PEM for further research.

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2. FUZZY PARAMETERIZED FUZZY SOFT MATRICES

In this section, we introduce the concept of *fpfs*-matrices and their basic properties that are first given in the first author's PhD dissertation [17].

Throughout this paper, let E be a parameter set, $F(E)$ be the set of all fuzzy sets over E , and $\mu \in F(E)$. Here, a fuzzy set is denoted by $\{\mu^{(x)}x : x \in E\}$ instead of $\{(x, \mu(x)) : x \in E\}$.

Definition 2.1. [6, 17] Let U be a universal set, $\mu \in F(E)$, and α be a function from μ to $F(U)$. Then, the set $\{(\mu^{(x)}x, \alpha(\mu^{(x)}x)) : x \in E\}$ being the graphic of α is called a fuzzy parameterized fuzzy soft set (*fpfs-set*) parameterized via E over U (or briefly over U).

In the present paper, the set of all *fpfs*-sets over U is denoted by $FPFS_E(U)$. In $FPFS_E(U)$, since the *graph*(α) and α generate each other uniquely, the notations are interchangeable. Therefore, unless it causes any confusion, we denote an *fpfs-set* *graph*(α) by α .

Example 2.1. Let $E = \{x_1, x_2, x_3, x_4\}$ and $U = \{u_1, u_2, u_3, u_4, u_5\}$. Then,

$$\alpha = \{(0.8x_1, \{0.9u_1, 0.5u_4\}), (0x_2, \{0.3u_2, 0.5u_3\}), (0.1x_3, \{0.7u_1, 0.8u_3, 0.6u_4\}), (x_4, \{u_3, 0.9u_5\})\}$$

is an *fpfs-set* over U .

Definition 2.2. [17] Let $\alpha \in FPFS_E(U)$. Then, $[a_{ij}]$ is called the matrix representation of α (or briefly *fpfs-matrix* of α) and is defined by

$$[a_{ij}] := \begin{bmatrix} a_{01} & a_{02} & a_{03} & \dots & a_{0n} & \dots \\ a_{11} & a_{12} & a_{13} & \dots & a_{1n} & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{0, 1, 2, \dots\}$ and $j \in \{1, 2, \dots\}$,

$$a_{ij} := \begin{cases} \mu(x_j), & i = 0 \\ \alpha(\mu^{(x_j)}x_j)(u_i), & i \neq 0 \end{cases}$$

Here, if $|U| = m - 1$ and $|E| = n$, then $[a_{ij}]$ has order $m \times n$.

From now on, the set of all *fpfs*-matrices parameterized via E over U is denoted by $FPFS_E[U]$.

Example 2.2. The *fpfs-matrix* of α provided in Example 2.1 is as follows:

$$[a_{ij}] = \begin{bmatrix} 0.8 & 0 & 0.1 & 1 \\ 0.9 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0.8 & 1 \\ 0.5 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Definition 2.3. Let $[a_{ij}] \in FPFS_E[U]$. For all i and j , if $a_{ij} = \lambda$, then $[a_{ij}]$ is called λ -*fpfs-matrix* and is denoted by $[\lambda]$. Here, $[0]$ is called empty *fpfs-matrix* and $[1]$ is called universal *fpfs-matrix*.

Definition 2.4. Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$, $I_E := \{j : x_j \in E\}$, and $R \subseteq I_E$. If

$$c_{ij} := \begin{cases} a_{ij}, & j \in R \\ b_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ is called *Rb-restriction* of $[a_{ij}]$ and is denoted by $[(a_Rb)_{ij}]$. Briefly, if $[b_{ij}] = [0]$, then $[(a_R)_{ij}]$ can be used instead of $[(a_{R0})_{ij}]$. It is clear that

$$(a_R)_{ij} = \begin{cases} a_{ij}, & j \in R \\ 0, & j \in I_E \setminus R \end{cases}$$

Example 2.3. Let $R = \{1, 3, 4\}$ and $S = \{2, 4\}$. Then, the *R1-* and *S-*restriction of $[a_{ij}]$ provided in Example 2.2 are as follows:

$$[(a_{R1})_{ij}] = \begin{bmatrix} 0.8 & 1 & 0.1 & 1 \\ 0.9 & 1 & 0.7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0.8 & 1 \\ 0.5 & 1 & 0.6 & 0 \\ 0 & 1 & 0 & 0.9 \end{bmatrix} \quad \text{and} \quad [(a_S)_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Definition 2.5. [17] Let $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. For all i and j , if $a_{ij} \leq b_{ij}$, then $[a_{ij}]$ is called a *submatrix* of $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\subseteq} [b_{ij}]$.

Proposition 2.1. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. Then,

- i. $[a_{ij}] \tilde{\subseteq} [1]$
- ii. $[0] \tilde{\subseteq} [a_{ij}]$
- iii. $[a_{ij}] \tilde{\subseteq} [a_{ij}]$
- iv. $([a_{ij}] \tilde{\subseteq} [b_{ij}] \wedge [b_{ij}] \tilde{\subseteq} [c_{ij}]) \Rightarrow [a_{ij}] \tilde{\subseteq} [c_{ij}]$

Definition 2.6. [17] Let $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. For all i and j , if $a_{ij} = b_{ij}$, then $[a_{ij}]$ and $[b_{ij}]$ are called *equal fpfs-matrices* and is denoted by $[a_{ij}] = [b_{ij}]$.

Definition 2.7. [17] Let $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. If $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ and $[a_{ij}] \neq [b_{ij}]$, then $[a_{ij}]$ is called a *proper submatrix* of $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\subset} [b_{ij}]$.

Proposition 2.2. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. Then,

- i. $([a_{ij}] = [b_{ij}] \wedge [b_{ij}] = [c_{ij}]) \Rightarrow [a_{ij}] = [c_{ij}]$
- ii. $([a_{ij}] \tilde{\subseteq} [b_{ij}] \wedge [b_{ij}] \tilde{\subseteq} [a_{ij}]) \Leftrightarrow [a_{ij}] = [b_{ij}]$

Definition 2.8. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. For all i and j , if $c_{ij} := \max\{a_{ij}, b_{ij}\}$, then $[c_{ij}]$ is called *union* of $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\cup} [b_{ij}]$.

Definition 2.9. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. For all i and j , if $c_{ij} := \min\{a_{ij}, b_{ij}\}$, then $[c_{ij}]$ is called *intersection* of $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\cap} [b_{ij}]$.

Example 2.4. Assume that two fpfs-matrices $[a_{ij}]$ and $[b_{ij}]$ are as follows:

$$[a_{ij}] = \begin{bmatrix} 0.3 & 0.7 & 0.1 & 1 \\ 0.5 & 0 & 0.8 & 0.6 \\ 1 & 0.1 & 0.6 & 0 \\ 0 & 0.5 & 0.1 & 1 \\ 0 & 0.2 & 0.7 & 0.9 \\ 0.4 & 1 & 0 & 0.4 \end{bmatrix} \quad \text{and} \quad [b_{ij}] = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.8 \\ 1 & 0 & 0.6 & 0.7 \\ 0.8 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.7 & 0.6 & 1 \\ 0.6 & 0.2 & 0.4 & 0.5 \\ 0.3 & 0.9 & 0.8 & 0.9 \end{bmatrix}$$

Then,

$$[a_{ij}] \tilde{\cup} [b_{ij}] = \begin{bmatrix} 0.9 & 0.7 & 0.7 & 1 \\ 1 & 0 & 0.8 & 0.7 \\ 1 & 0.5 & 0.6 & 0.2 \\ 0.4 & 0.7 & 0.6 & 1 \\ 0.6 & 0.2 & 0.7 & 0.9 \\ 0.4 & 1 & 0.8 & 0.9 \end{bmatrix} \quad \text{and} \quad [a_{ij}] \tilde{\cap} [b_{ij}] = \begin{bmatrix} 0.3 & 0.5 & 0.1 & 0.8 \\ 0.5 & 0 & 0.6 & 0.6 \\ 0.8 & 0.1 & 0.5 & 0 \\ 0 & 0.5 & 0.1 & 1 \\ 0 & 0.2 & 0.4 & 0.5 \\ 0.3 & 0.9 & 0 & 0.4 \end{bmatrix}$$

Proposition 2.3. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFSE[U]$. Then,

- i. $[a_{ij}] \tilde{\cup} [a_{ij}] = [a_{ij}]$ and $[a_{ij}] \tilde{\cap} [a_{ij}] = [a_{ij}]$
- ii. $[a_{ij}] \tilde{\cup} [0] = [a_{ij}]$ and $[a_{ij}] \tilde{\cap} [0] = [0]$
- iii. $[a_{ij}] \tilde{\cup} [1] = [1]$ and $[a_{ij}] \tilde{\cap} [1] = [a_{ij}]$
- iv. $[a_{ij}] \tilde{\cup} [b_{ij}] = [b_{ij}] \tilde{\cup} [a_{ij}]$ and $[a_{ij}] \tilde{\cap} [b_{ij}] = [b_{ij}] \tilde{\cap} [a_{ij}]$
- v. $([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cup} [c_{ij}] = [a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cup} [c_{ij}])$ and $([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cap} [c_{ij}] = [a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cap} [c_{ij}])$
- vi. $[a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cap} [c_{ij}]) = ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}])$ and $[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cup} [c_{ij}]) = ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}])$

Proof.

$$\begin{aligned} [a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cap} [c_{ij}]) &= [a_{ij}] \tilde{\cup} [\min\{b_{ij}, c_{ij}\}] \\ &= [\max\{a_{ij}, \min\{b_{ij}, c_{ij}\}\}] \\ &= [\min\{\max\{a_{ij}, b_{ij}\}, \max\{a_{ij}, c_{ij}\}\}] \\ &= [\max\{a_{ij}, b_{ij}\}] \tilde{\cap} [\max\{a_{ij}, c_{ij}\}] \\ &= ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}]) \end{aligned}$$

□

The proofs of the others can be performed similarly.

Definition 2.10. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFSE[U]$. For all i and j , if $c_{ij} := \max\{0, a_{ij} - b_{ij}\}$, then $[c_{ij}]$ is called difference between $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\setminus} [b_{ij}]$.

Proposition 2.4. [17] Let $[a_{ij}], [b_{ij}] \in FPFSE[U]$. Then,

- i. $[a_{ij}] \tilde{\setminus} [a_{ij}] = [0]$
- ii. $[a_{ij}] \tilde{\setminus} [0] = [a_{ij}]$
- iii. $[a_{ij}] \tilde{\setminus} [1] = [0]$
- iv. $([a_{ij}] \tilde{\setminus} [b_{ij}] = [0]) \Rightarrow [a_{ij}] \tilde{\subseteq} [b_{ij}]$

Remark 2.1. It must be noted that the difference operation is non-commutative and non-associative.

Definition 2.11. [17] Let $[a_{ij}], [b_{ij}] \in FPFSE[U]$. For all i and j , if $b_{ij} := 1 - a_{ij}$, then $[b_{ij}]$ is complement of $[a_{ij}]$ and is denoted by $[a_{ij}]^{\tilde{c}}$ or $[a_{ij}^{\tilde{c}}]$.

Proposition 2.5. [17] Let $[a_{ij}] \in FPFSE[U]$. Then,

- i. $([a_{ij}]^{\tilde{c}})^{\tilde{c}} = [a_{ij}]$
- ii. $[0]^{\tilde{c}} = [1]$

Proposition 2.6. [17] Let $[a_{ij}], [b_{ij}] \in FPFSE[U]$. Then, the following De Morgan's laws are valid.

- i. $([a_{ij}] \tilde{\cup} [b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}} \tilde{\cap} [b_{ij}]^{\tilde{c}}$
- ii. $([a_{ij}] \tilde{\cap} [b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}} \tilde{\cup} [b_{ij}]^{\tilde{c}}$

Proof.

$$\begin{aligned}
 ([a_{ij}] \tilde{\cup} [b_{ij}])^{\tilde{c}} &= [\max\{a_{ij}, b_{ij}\}]^{\tilde{c}} \\
 &= [1 - \max\{a_{ij}, b_{ij}\}] \\
 &= [\min\{1 - a_{ij}, 1 - b_{ij}\}] \\
 &= [a_{ij}]^{\tilde{c}} \tilde{\cap} [b_{ij}]^{\tilde{c}}
 \end{aligned}$$

□

The proof of *ii.* can be performed similarly.

Definition 2.12. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFSE[U]$. For all i and j , if $c_{ij} := |a_{ij} - b_{ij}|$, then $[c_{ij}]$ is called symmetric difference between $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\Delta} [b_{ij}]$.

Definition 2.13. [17] Let $[a_{ij}], [b_{ij}] \in FPFSE[U]$. If $[a_{ij}] \tilde{\cap} [b_{ij}] = [0]$, then $[a_{ij}]$ and $[b_{ij}]$ are called disjoint.

Example 2.5. Let us consider the fpfs-matrices $[a_{ij}]$ and $[b_{ij}]$ provided in Example 2.4. Then,

$$[a_{ij}] \tilde{\setminus} [b_{ij}] = \begin{bmatrix} 0 & 0.2 & 0 & 0.2 \\ 0 & 0 & 0.2 & 0 \\ 0.2 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0.4 \\ 0.1 & 0.1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad [a_{ij}] \tilde{\Delta} [b_{ij}] = \begin{bmatrix} 0.6 & 0.2 & 0.6 & 0.2 \\ 0.5 & 0 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.1 & 0.2 \\ 0.4 & 0.2 & 0.5 & 0 \\ 0.6 & 0 & 0.3 & 0.4 \\ 0.1 & 0.1 & 0.8 & 0.5 \end{bmatrix}$$

Definition 2.14. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFSE[U]$, $I_E := \{j : x_j \in E\}$, and $R \subseteq I_E$. If

$$c_{ij} := \begin{cases} \max\{a_{ij}, \min_{k \in R}\{b_{ik}\}\}, & j \in R \\ a_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ is called R -relative union of $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\cup}_R^r [b_{ij}]$. Here, for the brevity, “relative union” can be used instead of “ I_E -relative union” and can be denoted by $[a_{ij}] \tilde{\cup}^r [b_{ij}]$.

Definition 2.15. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFSE[U]$, $I_E := \{j : x_j \in E\}$, and $R \subseteq I_E$. If

$$c_{ij} := \begin{cases} \min\{a_{ij}, \max_{k \in R}\{b_{ik}\}\}, & j \in R \\ a_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ is called R -relative intersection of $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\cap}_R^r [b_{ij}]$. Here, for brevity, “relative intersection” can be used instead of “ I_E -relative intersection” and can be denoted by $[a_{ij}] \tilde{\cap}^r [b_{ij}]$.

Definition 2.16. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFSE[U]$, $I_E := \{j : x_j \in E\}$, and $R \subseteq I_E$. If

$$c_{ij} := \begin{cases} \max\{0, a_{ij} - \min_{k \in R}\{b_{ik}\}\}, & j \in R \\ a_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ is called R -relative difference between $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}] \tilde{\setminus}_R^r [b_{ij}]$. Here, for brevity, “relative difference” can be used instead of “ I_E -relative difference” and can be denoted by $[a_{ij}] \tilde{\setminus}^r [b_{ij}]$.

Example 2.6. Let us consider the *fpfs*-matrices $[a_{ij}]$ and $[b_{ij}]$ provided in Example 2.4 and let $R = \{2, 3, 4\}$. Then,

$$[a_{ij}] \tilde{\cup}_R^r [b_{ij}] = \begin{bmatrix} 0.5 & 0.7 & 0.5 & 1 \\ 0.5 & 0 & 0.8 & 0.6 \\ 1 & 0.2 & 0.6 & 0.2 \\ 0.4 & 0.5 & 0.4 & 1 \\ 0.2 & 0.2 & 0.7 & 0.9 \\ 0.4 & 1 & 0.3 & 0.4 \end{bmatrix} \quad \text{and} \quad [a_{ij}] \tilde{\cap}_R^r [b_{ij}] = \begin{bmatrix} 0.3 & 0.7 & 0.1 & 0.8 \\ 0.5 & 0 & 0.7 & 0.6 \\ 1 & 0.1 & 0.5 & 0 \\ 0 & 0.5 & 0.1 & 1 \\ 0 & 0.2 & 0.5 & 0.5 \\ 0.4 & 0.9 & 0 & 0.4 \end{bmatrix}$$

Proposition 2.7. [17] Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFSE[U]$. Then,

- i. $[a_{ij}] \tilde{\cup}_R^r [0] = [a_{ij}]$ and $[0] \tilde{\cap}_R^r [a_{ij}] = [0]$
- ii. $[1] \tilde{\cup}_R^r [a_{ij}] = [1]$ and $[a_{ij}] \tilde{\cap}_R^r [1] = [a_{ij}]$
- iii. $([a_{ij}] \tilde{\cup}_R^r [b_{ij}]) \tilde{\cup}_R^r [c_{ij}] = [a_{ij}] \tilde{\cup}_R^r ([b_{ij}] \tilde{\cup}_R^r [c_{ij}])$ and $([a_{ij}] \tilde{\cap}_R^r [b_{ij}]) \tilde{\cap}_R^r [c_{ij}] = [a_{ij}] \tilde{\cap}_R^r ([b_{ij}] \tilde{\cap}_R^r [c_{ij}])$

Remark 2.2. It must be noted that the relative intersection and relative union of *fpfs*-matrices are non-commutative and non-distributive.

Proposition 2.8. [17] Let $[a_{ij}], [b_{ij}] \in FPFSE[U]$. Then, the following De Morgan's laws are valid.

- i. $([a_{ij}] \tilde{\cup}_R^r [b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}} \tilde{\cap}_R^r [b_{ij}]^{\tilde{c}}$
- ii. $([a_{ij}] \tilde{\cap}_R^r [b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}} \tilde{\cup}_R^r [b_{ij}]^{\tilde{c}}$

Proof. If $j \in R \subseteq I_E$, then

$$\begin{aligned} ([a_{ij}] \tilde{\cup}_R^r [b_{ij}])^{\tilde{c}} &= [\max\{a_{ij}, \min_{k \in R}\{b_{ik}\}\}]^{\tilde{c}} \\ &= [1 - \max\{a_{ij}, \min_{k \in R}\{b_{ik}\}\}] \\ &= [\min\{1 - a_{ij}, 1 - \min_{k \in R}\{b_{ik}\}\}] \\ &= [\min\{1 - a_{ij}, \max_{k \in R}\{1 - b_{ik}\}\}] \\ &= [a_{ij}]^{\tilde{c}} \tilde{\cap}_R^r [b_{ij}]^{\tilde{c}} \end{aligned}$$

and if $j \in I_E \setminus R$, then

$$\begin{aligned} ([a_{ij}] \tilde{\cup}_R^r [b_{ij}])^{\tilde{c}} &= [a_{ij}]^{\tilde{c}} \\ &= [1 - a_{ij}] \\ &= [a_{ij}]^{\tilde{c}} \tilde{\cap}_R^r [b_{ij}]^{\tilde{c}} \end{aligned}$$

□

The proof of ii. can be made in a similar way.

Definition 2.17. Let $[a_{ij}]_{m \times n_1} \in FPFSE_1[U]$, $[b_{ik}]_{m \times n_2} \in FPFSE_2[U]$, and $[c_{ip}]_{m \times n_1 n_2} \in FPFSE_{E_1 \times E_2}[U]$ such that $p = n_2(j - 1) + k$. For all i and p , if $c_{ip} := \min\{a_{ij}, b_{ik}\}$, then $[c_{ip}]$ is called *and-product* of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \wedge [b_{ik}]$.

Definition 2.18. Let $[a_{ij}]_{m \times n_1} \in FPFSE_1[U]$, $[b_{ik}]_{m \times n_2} \in FPFSE_2[U]$, and $[c_{ip}]_{m \times n_1 n_2} \in FPFSE_{E_1 \times E_2}[U]$ such that $p = n_2(j - 1) + k$. For all i and p , if $c_{ip} := \max\{a_{ij}, b_{ik}\}$, then $[c_{ip}]$ is called *or-product* of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \vee [b_{ik}]$.

Definition 2.19. Let $[a_{ij}]_{m \times n_1} \in FPFSE_1[U]$, $[b_{ik}]_{m \times n_2} \in FPFSE_2[U]$, and $[c_{ip}]_{m \times n_1 n_2} \in FPFSE_{E_1 \times E_2}[U]$ such that $p = n_2(j - 1) + k$. For all i and p , if $c_{ip} := \min\{a_{ij}, 1 - b_{ik}\}$, then $[c_{ip}]$ is called *andnot-product* of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \bar{\wedge} [b_{ik}]$.

Definition 2.20. Let $[a_{ij}]_{m \times n_1} \in FPFSE_1[U]$, $[b_{ik}]_{m \times n_2} \in FPFSE_2[U]$, and $[c_{ip}]_{m \times n_1 n_2} \in FPFSE_{E_1 \times E_2}[U]$ such that $p = n_2(j - 1) + k$. For all i and p , if $c_{ip} := \max\{a_{ij}, 1 - b_{ik}\}$, then $[c_{ip}]$ is called *ornot-product* of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \bar{\vee} [b_{ik}]$.

Example 2.7. Let us consider the *fpfs*-matrices $[a_{ij}]$ and $[b_{ik}]$ provided in Example 2.4. Then, $[a_{ij}] \vee [b_{ik}]$ is as follows:

$$\begin{bmatrix} 0.3 & 0.5 & 0.3 & 0.3 & 0.7 & 0.7 & 0.7 & 0.7 & 0.1 & 0.5 & 0.3 & 0.2 & 1 & 1 & 1 & 1 \\ 0.5 & 1 & 0.5 & 0.5 & 0 & 1 & 0.4 & 0.3 & 0.8 & 1 & 0.8 & 0.8 & 0.6 & 1 & 0.6 & 0.6 \\ 1 & 1 & 1 & 1 & 0.2 & 0.5 & 0.5 & 0.8 & 0.6 & 0.6 & 0.6 & 0.8 & 0.2 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.3 & 0.4 & 0 & 0.6 & 0.5 & 0.5 & 0.5 & 0.6 & 0.3 & 0.4 & 0.1 & 1 & 1 & 1 & 1 \\ 0.4 & 0.8 & 0.6 & 0.5 & 0.4 & 0.8 & 0.6 & 0.5 & 0.7 & 0.8 & 0.7 & 0.7 & 0.9 & 0.9 & 0.9 & 0.9 \\ 0.7 & 0.4 & 0.4 & 0.4 & 1 & 1 & 1 & 1 & 0.7 & 0.1 & 0.2 & 0.1 & 0.7 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Proposition 2.9. Let $[a_{ij}]$, $[b_{ik}]$, $[c_{il}]$ be three *fpfs*-matrices over U . Then,

$$([a_{ij}] \wedge [b_{ik}]) \wedge [c_{il}] = [a_{ij}] \wedge ([b_{ik}] \wedge [c_{il}]) \text{ and } ([a_{ij}] \vee [b_{ik}]) \vee [c_{il}] = [a_{ij}] \vee ([b_{ik}] \vee [c_{il}])$$

Remark 2.3. It must be noted that the products mentioned above of *fpfs*-matrices are non-commutative and non-distributive.

3. PREVALENCE EFFECT METHOD (PEM)

In this section, we propose a new soft decision-making method called Prevalence Effect Method (PEM).

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m \times n}$ such that $i \in \{0, 1, 2, \dots, m-1\}$, $j \in \{1, 2, \dots, n\}$, $m \geq 2$, and $n \geq 1$

Step 2. Obtain a matrix $[s_{i1}]$ defined by $s_{i1} := \sum_{j=1}^n \left[\left(\frac{1}{m-1} \sum_{k=1}^{m-1} a_{kj} \right) \left(\frac{1}{n} \sum_{t=1}^n a_{it} \right) a_{0j} a_{ij} \right]$ such that $i \in \{1, 2, \dots, m-1\}$

Step 3. Obtain a decision set $\left\{ \frac{s_{k1}}{\max s_{i1}} u_k \mid u_k \in U \right\}$

Here, a_{ij} shows to what extent i^{th} alternative provide the j^{th} parameter such that $i \neq 0$, a_{0j} shows how essential j^{th} parameter is for the user, $\frac{1}{n} \sum_{t=1}^n a_{it}$ refers to the prevalence effect value of i^{th} alternative, $\frac{1}{m-1} \sum_{k=1}^{m-1} a_{kj}$ to the prevalence effect value of j^{th} parameter, and s_{i1} to the score value.

4. AN APPLICATION OF PEM

In this section, we apply PEM to a real problem in image denoising. Image denoising (noise removal), which is a preprocess in image processing, positively affects the success rate of other procedures. Therefore, a great many studies have been conducted in this area [44–48].

We, in this study, consider five noise removal methods - Progressive Switching Median Filter (PSMF) [44], Decision-Based Algorithm (DBA) [45], Modified Decision-Based Unsymmetrical Trimmed Median Filter (MDBUTMF) [46], Noise Adaptive Fuzzy Switching Median Filter (NAFSMF) [47], and Different Applied Median Filter (DAMF) [48] - used in [48] for salt-and-pepper noise removal, which is a kind of impulse noise. We compare these methods with regard to performance by using 15 traditional images (Cameraman, Lena, Peppers, Baboon, Plane, Bridge, Pirate, Elaine, Boat, Lake, Flintstones, Living Room, House, Parrot, and Hill) with 512×512 pixels and 40 test images with 600×600 pixels in the TEST IMAGES Database [49], ranging in noise densities from 10% to 90%, and two image quality metrics: Peak Signal to Noise Ratio (PSNR) and Structural Similarity (SSIM) [50]. The results in Table 1 and 2 show that DAMF outperforms the others in any noise density.

TABLE 1. The mean SSIM results for the 15 traditional images

Algorithm	10%	20%	30%	40%	50%	60%	70%	80%	90%
PSMF	0.9028	0.8715	0.8018	0.6988	0.4903	0.1882	0.0633	0.0318	0.0139
DBA	0.9079	0.8664	0.8097	0.7376	0.6521	0.5552	0.4567	0.3623	0.2937
MDBUTMF	0.8841	0.7994	0.7443	0.7657	0.7963	0.7880	0.7501	0.6443	0.3052
NAFSMF	0.9147	0.8916	0.8669	0.8409	0.8124	0.7796	0.7403	0.6872	0.5736
DAMF	0.9253	0.9113	0.8946	0.8752	0.8523	0.8244	0.7892	0.7398	0.6572

TABLE 2. The mean SSIM results for the 40 test images

Algorithm	10%	20%	30%	40%	50%	60%	70%	80%	90%
PSMF	0.9444	0.9014	0.843	0.7435	0.5663	0.2822	0.0817	0.0383	0.0171
DBA	0.9798	0.9467	0.8964	0.8247	0.7316	0.6218	0.5002	0.3794	0.2998
MDBUTMF	0.9431	0.8349	0.7724	0.8154	0.8748	0.8813	0.8489	0.7407	0.3730
NAFSMF	0.9790	0.9602	0.9411	0.9209	0.8988	0.8724	0.8385	0.7889	0.6648
DAMF	0.9911	0.9819	0.9705	0.9563	0.9392	0.9174	0.8885	0.8451	0.7595

Suppose that the success in high noise densities is more important than in the others. In that case, the values in Table 1 can be represented with an *fpfs*-matrix as follows:

$$[a_{ij}] := \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\ 0.9028 & 0.8715 & 0.8018 & 0.6988 & 0.4903 & 0.1882 & 0.0633 & 0.0318 & 0.0139 \\ 0.9079 & 0.8664 & 0.8097 & 0.7376 & 0.6521 & 0.5552 & 0.4567 & 0.3623 & 0.2937 \\ 0.8841 & 0.7994 & 0.7443 & 0.7657 & 0.7963 & 0.7880 & 0.7501 & 0.6443 & 0.3052 \\ 0.9147 & 0.8916 & 0.8669 & 0.8409 & 0.8124 & 0.7796 & 0.7403 & 0.6872 & 0.5736 \\ 0.9253 & 0.9113 & 0.8946 & 0.8752 & 0.8523 & 0.8244 & 0.7892 & 0.7398 & 0.6572 \end{bmatrix}$$

If we apply PEM to the $[a_{ij}]$, then the score matrix and the decision set are as follows:

$$[s_{i1}] = [0.2160 \ 0.5171 \ 0.7395 \ 0.8957 \ 1]^T$$

and

$$\{^{0.2160}\text{PSMF}, ^{0.5171}\text{DBA}, ^{0.7395}\text{MDBUTMF}, ^{0.8957}\text{NAFSMF}, ^1\text{DAMF}\}$$

The scores show that DAMF outperforms the other methods and the order DAMF, NAFSMF, MDBUTMF, DBA, and PSMF is valid.

Similarly, the values in Table 2 can be represented with an *fpfs*-matrix as follows:

$$[b_{ij}] := \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\ 0.9444 & 0.9014 & 0.843 & 0.7435 & 0.5663 & 0.2822 & 0.0817 & 0.0383 & 0.0171 \\ 0.9798 & 0.9467 & 0.8964 & 0.8247 & 0.7316 & 0.6218 & 0.5002 & 0.3794 & 0.2998 \\ 0.9431 & 0.8349 & 0.7724 & 0.8154 & 0.8748 & 0.8813 & 0.8489 & 0.7407 & 0.3730 \\ 0.9790 & 0.9602 & 0.9411 & 0.9209 & 0.8988 & 0.8724 & 0.8385 & 0.7889 & 0.6648 \\ 0.9911 & 0.9819 & 0.9705 & 0.9563 & 0.9392 & 0.9174 & 0.8885 & 0.8451 & 0.7595 \end{bmatrix}$$

If we apply PEM to the $[b_{ij}]$, then the score matrix and the decision set are as follows:

$$[s_{i1}] = [0.2114 \ 0.5026 \ 0.7291 \ 0.8999 \ 1]^T$$

and

$$\{^{0.2114}\text{PSMF}, ^{0.5026}\text{DBA}, ^{0.7291}\text{MDBUTMF}, ^{0.8999}\text{NAFSMF}, ^1\text{DAMF}\}$$

The scores show that DAMF outperforms the other methods and the order DAMF, NAFSMF, MDBUTMF, DBA, and PSMF is valid.

5. CONCLUSION

In this paper, we define the concept of *fpfs*-matrices. We then suggest a new method referred to as PEM. Afterwards, we successfully apply PEM to the determination of the performance of the methods used in [48]. It is clear that PEM, which is a fast and simple method, can be successfully applied to decision-making problems in various areas, such as machine learning and image processing. We also believe that the configuration of the other methods in the literature via *fpfs*-matrices is worth studying.

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