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Soft matrix theory and its decision making

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ABSTRACT

In this work, we define soft matrices and their operations which are more functional to make theoretical studies in the soft set theory. We then define products of soft matrices and their properties. We finally construct a soft max–min decision making method which can be successfully applied to the problems that contain uncertainties.

1. Introduction

Soft set theory [1] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In [1–4], Molodtsov successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement and so on.

By using the rough sets [5], Maji et al. [6,7] presented an application of soft sets in a decision making problem and published a detailed theoretical study on soft sets. Chen et al. [8] and Xiao et al. [9] presented research on synthetically evaluating a method for business competitive capacity based on soft sets. Chen et al. [10,8] and Kong et al. [11] introduced a new definition of soft set parameterization reduction. Xiao et al. [12] and Pei and Miao [13] discussed the relationship between soft sets and information systems. Mushrif et al. [14] presented a new algorithm for classification of the natural textures. The proposed classification algorithm is based on the notions of soft set theory.

The algebraic structure of soft set theories has been studied increasingly in recent years. Aktaş and Çağman [15] gave a definition of soft groups. Jun [16] introduced the notion of soft BCK/BCI-algebras and soft subalgebras. Jun and Park [17] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. Park et al. [18] introduced the notion of soft WS-algebras and then derived their basic properties. Feng et al. [19] initiated the study of soft semirings by using the soft set theory and investigated several related properties. Sun et al. [20] introduced a basic version of soft module theory, which extends the notion of a module by including some algebraic structures in soft sets. Zou and Xiao [21] presented data analysis approaches of soft sets under incomplete information. These approaches presented in [21] are preferable for reflecting actual states of incomplete data in soft sets.

Maji et al. [22] defined the fuzzy soft sets. Afterwards, many researchers have worked on this concept. Aktaş and Çağman [15] also compared soft sets to the related concepts of fuzzy sets and rough sets, providing examples to clarify their differences. Roy and Maji [23] presented some results on an application of fuzzy soft sets in decision making problems. Yang et al. [24] defined the reduction of fuzzy soft sets and then analyzed a decision making problem by fuzzy soft sets. Majumdar and Samanta [25] introduced several similarity measures of fuzzy soft sets. Kong et al. [11] and Xiao et al. [26] presented a working of some approximations based on soft sets.

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Based on the theory of soft sets, the analysis was developed in [4], and the notions of soft number, soft derivative, soft integral, etc. are formulated. This technique is applied to soft optimization problems by Kovkov et al. [27]. Based on the analysis of several operations on soft sets introduced by Ali et al. [28]. Çağman and Enginoğlu [29] redefined the operations of Molodtsov's soft sets to make them more functional for improving several new results. By using these new definitions they then construct soft decision making methods.

Up to the present, the applications of soft set theory generally solve problems with the help of the rough sets or fuzzy soft sets. In this paper, we first define soft matrices which are representations of soft sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the soft sets represented by them in a computer. Here, we also construct a soft decision making which is more practical and can be successfully applied to many problems that contain uncertainties without using the rough sets and fuzzy soft sets.

The presentation of the rest of this paper is organized as follows. In the next section, we define soft matrices and their operations. In Section 3, we define *And*-product, *Or*-product *And–Not*-product and *Or–Not*-product of soft matrices and their properties. In Section 4, we first define a soft max–min decision function and then a soft max–min decision making method. In Section 5, we give an example which shows that these methods successfully work. In the final section, some concluding comments are presented.

2. Soft matrices

In this section, we define soft matrices which are representative of the soft sets. This style of representation is useful for storing a soft set in computer memory. The operations can be presented by the matrices which are very useful and applicable.

Definition 1 (*[1]*)**.** Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and $A \subseteq E$. A *soft set* (f_A, E) on the universe U is defined by the set of ordered pairs

$$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$$

where $f_A : E \to P(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$.

Here, f_A is called an *approximate function* of the soft set (f_A, E). The set $f_A(e)$ is called *e-approximate value set* or *e-approximate set* which consists of related objects of the parameter $e \in E$.

Definition 2. Let (f_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by

$$R_A = \{(u, e) : e \in A, u \in f_A(e)\}$$

which is called a *relation form* of (f_A, E). The characteristic function of R_A is written by

$$\chi_{R_A} : U \times E \to \{0, 1\}, \qquad \chi_{R_A}(u, e) = \begin{cases} 1, & (u, e) \in R_A \\ 0, & (u, e) \notin R_A. \end{cases}$$

If $U = \{u_1, u_2, \dots, u_m\}$, $E = \{e_1, e_2, \dots, e_n\}$ and $A \subseteq E$, then the R_A can be presented by a table as in the following form

R_A	e_1	e_2	$\cdots$	e_n
u_1	$\chi_{R_A}(u_1, e_1)$	$\chi_{R_A}(u_1, e_2)$	$\cdots$	$\chi_{R_A}(u_1, e_n)$
u_2	$\chi_{R_A}(u_2, e_1)$	$\chi_{R_A}(u_2, e_2)$	$\cdots$	$\chi_{R_A}(u_2, e_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\chi_{R_A}(u_m, e_1)$	$\chi_{R_A}(u_m, e_2)$	$\cdots$	$\chi_{R_A}(u_m, e_n)$

If $a_{ij} = \chi_{R_A}(u_i, e_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

which is called an $m \times n$ soft matrix of the soft set (f_A, E) over U.

According to this definition, a soft set (f_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. It means that a soft set (f_A, E) is formally equal to its soft matrix $[a_{ij}]_{m\times n}$. Therefore, we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

The set of all $m \times n$ soft matrices over U will be denoted by $SM_{m\times n}$. From now on we shall delete the subscripts $m \times n$ of $[a_{ij}]_{m\times n}$, we use $[a_{ij}]$ instead of $[a_{ij}]_{m\times n}$, since $[a_{ij}] \in SM_{m\times n}$ means that $[a_{ij}]$ is an $m \times n$ soft matrix for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$.

Example 1. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters. If $A = \{e_2, e_3, e_4\}$ and $f_A(e_2) = \{u_2, u_4\}, f_A(e_3) = \emptyset, f_A(e_4) = U$, then we write a soft set

$$(f_A, E) = \{(e_2, \{u_2, u_4\}), (e_4, U)\}$$

and then the relation form of (f_A, E) is written by

$$R_A = \{(u_2, e_2), (u_4, e_2), (u_1, e_4), (u_2, e_4), (u_3, e_4), (u_4, e_4), (u_5, e_4)\}.$$

Hence, the soft matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Definition 3. Let $[a_{ij}] \in SM_{m\times n}$. Then $[a_{ij}]$ is called

(a) a zero soft matrix, denoted by [0], if $a_{ij} = 0$ for all i and j.
(b) an A-universal soft matrix, denoted by $[\tilde{a}_{ij}]$, if $a_{ij} = 1$ for all $j \in I_A = \{j : e_j \in A\}$ and i.
(c) a universal soft matrix, denoted by [1], if $a_{ij} = 1$ for all i and j.

Example 2. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set, $E = \{e_1, e_2, e_3, e_4\}$ be a set of parameters, and $[a_{ij}], [b_{ij}], [c_{ij}] \in SM_{5\times 4}$.

If $A = \{e_1, e_3\}$ and $f_A(e_1) = \emptyset, f_A(e_3) = \emptyset$, then $[a_{ij}] = [0]$ is a zero soft matrix written by

$$[0] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

If $B = \{e_1, e_2\}$ and $f_B(e_1) = U, f_B(e_2) = U$, then $[b_{ij}] = [\tilde{b}_{ij}]$ is a B-universal soft matrix written by

$$[\tilde{b}_{ij}] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

If $C = E$, and $f_C(e_i) = U$ for all $e_i \in C, i = 1, 2, 3, 4$, then $[c_{ij}] = [1]$ is a universal soft matrix written by

$$[1] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Definition 4. Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then

(a) $[a_{ij}]$ is a soft submatrix of $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\subseteq} [b_{ij}]$, if $a_{ij} \leqslant b_{ij}$ for all i and j.
(b) $[a_{ij}]$ is a proper soft submatrix of $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\subset} [b_{ij}]$, If $a_{ij} \leqslant b_{ij}$ for at least one term $a_{ij} < b_{ij}$ for all i and j.
(c) $[a_{ij}]$ and $[b_{ij}]$ are soft equal matrices, denoted by $[a_{ij}] = [b_{ij}]$, if $a_{ij} = b_{ij}$ for all i and j.

Definition 5. Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then the soft matrix $[c_{ij}]$ is called

(a) union of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}] \tilde{\cup} [b_{ij}]$, if $c_{ij} = \max\{a_{ij}, b_{ij}\}$ for all i and j.
(b) intersection of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}] \tilde{\cap} [b_{ij}]$, if $c_{ij} = \min\{a_{ij}, b_{ij}\}$ for all i and j.
(c) complement of $[a_{ij}]$, denoted by $[a_{ij}]^{\circ}$, if $c_{ij} = 1 - a_{ij}$ for all i and j.

Definition 6. Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then $[a_{ij}]$ and $[b_{ij}]$ are disjoint, if $[a_{ij}] \tilde{\cap} [b_{ij}] = [0]$ for all i and j.

Example 3. Assume that $[a_{ij}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $[b_{ij}] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.Then

$$[a_{ij}] \tilde{\cup} [b_{ij}] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad [a_{ij}] \tilde{\cap} [b_{ij}] = [0], \qquad [a_{ij}]^\circ = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Proposition 1. *Let $[a_{ij}] \in SM_{m\times n}$. Then*

i. $[[a_{ij}]^\circ]^\circ = [a_{ij}]$
ii. $[0]^\circ = [1]$.

Proposition 2. *Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then*

i. $[a_{ij}] \tilde{\subseteq} [1]$
ii. $[0] \tilde{\subseteq} [a_{ij}]$
iii. $[a_{ij}] \tilde{\subseteq} [a_{ij}]$
iv. $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ *and* $[b_{ij}] \tilde{\subseteq} [c_{ij}] \Rightarrow [a_{ij}] \tilde{\subseteq} [c_{ij}]$.

Proposition 3. *Let $[a_{ij}], [b_{ij}], [c_{ij}] \in SM_{m\times n}$. Then*

i. $[a_{ij}] = [b_{ij}]$ *and* $[b_{ij}] = [c_{ij}] \Leftrightarrow [a_{ij}] = [c_{ij}]$
ii. $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ *and* $[b_{ij}] \tilde{\subseteq} [a_{ij}] \Leftrightarrow [a_{ij}] = [b_{ij}]$.

Proposition 4. *Let $[a_{ij}], [b_{ij}], [c_{ij}] \in SM_{m\times n}$. Then*

i. $[a_{ij}] \tilde{\cup} [a_{ij}] = [a_{ij}]$
ii. $[a_{ij}] \tilde{\cup} [0] = [a_{ij}]$
iii. $[a_{ij}] \tilde{\cup} [1] = [1]$
iv. $[a_{ij}] \tilde{\cup} [a_{ij}]^\circ = [1]$
v. $[a_{ij}] \tilde{\cup} [b_{ij}] = [b_{ij}] \tilde{\cup} [a_{ij}]$
vi. $([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cup} [c_{ij}] = [a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cup} [c_{ij}])$.

Proposition 5. *Let $[a_{ij}], [b_{ij}], [c_{ij}] \in SM_{m\times n}$. Then*

i. $[a_{ij}] \tilde{\cap} [a_{ij}] = [a_{ij}]$
ii. $[a_{ij}] \tilde{\cap} [0] = [0]$
iii. $[a_{ij}] \tilde{\cap} [1] = [a_{ij}]$
iv. $[a_{ij}] \tilde{\cap} [a_{ij}]^\circ = [0]$
v. $[a_{ij}] \tilde{\cap} [b_{ij}] = [b_{ij}] \tilde{\cap} [a_{ij}]$
vi. $([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cap} [c_{ij}] = [a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cap} [c_{ij}])$.

Proposition 6. *Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then De Morgan's laws are valid*

i. $([a_{ij}] \tilde{\cup} [b_{ij}])^\circ = [a_{ij}]^\circ \tilde{\cap} [b_{ij}]^\circ$
ii. $([a_{ij}] \tilde{\cap} [b_{ij}])^\circ = [a_{ij}]^\circ \tilde{\cup} [b_{ij}]^\circ$.

Proof. For all i and j,

i.

$$\begin{aligned} ([a_{ij}] \tilde{\cup} [b_{ij}])^\circ &= [\max\{a_{ij}, b_{ij}\}]^\circ \\ &= [1 - \max\{a_{ij}, b_{ij}\}] \\ &= [\min\{1 - a_{ij}, 1 - b_{ij}\}] \\ &= [a_{ij}]^\circ \tilde{\cap} [b_{ij}]^\circ. \end{aligned}$$

ii. It can be proved similarly.

Example 4. Let $[a_{ij}], [b_{ij}] \in SM_{5\times4}$ as in the Example 3. Then

$$([a_{ij}] \tilde{\cup} [b_{ij}])^\circ = [a_{ij}]^\circ \tilde{\cap} [b_{ij}]^\circ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

and

$$([a_{ij}] \tilde{\cap} [b_{ij}])^\circ = [a_{ij}]^\circ \tilde{\cup} [b_{ij}]^\circ = [1].$$

Proposition 7. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in SM_{m\times n}$. *Then*

i. $[a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cap} [c_{ij}]) = ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}])$
ii. $[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cup} [c_{ij}]) = ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}])$.

3. Products of soft matrices

In this section, we define four special products of soft matrices to construct soft decision making methods.

Definition 7. Let $[a_{ij}], [b_{ik}] \in SM_{m\times n}$. Then *And*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\wedge : SM_{m\times n} \times SM_{m\times n} \to SM_{m\times n^2}, \qquad [a_{ij}] \wedge [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \min\{a_{ij}, b_{ik}\}$ such that $p = n(j-1) + k$.

Definition 8. Let $[a_{ij}], [b_{ik}] \in SM_{m\times n}$. Then *Or*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\vee : SM_{m\times n} \times SM_{m\times n} \to SM_{m\times n^2}, \qquad [a_{ij}] \vee [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \max\{a_{ij}, b_{ik}\}$ such that $p = n(j-1) + k$.

Definition 9. Let $[a_{ij}], [b_{ik}] \in SM_{m\times n}$. Then *And–Not*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\barwedge : SM_{m\times n} \times SM_{m\times n} \to SM_{m\times n^2}, \qquad [a_{ij}] \barwedge [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \min\{a_{ij}, 1 - b_{ik}\}$ such that $p = n(j-1) + k$.

Definition 10. Let $[a_{ij}], [b_{ik}] \in SM_{m\times n}$. Then *Or–Not*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\veebar : SM_{m\times n} \times SM_{m\times n} \to SM_{m\times n^2}, \qquad [a_{ij}] \veebar [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \max\{a_{ij}, 1 - b_{ik}\}$ such that $p = n(j-1) + k$.

Example 5. Assume that $[a_{ij}], [b_{ik}] \in SM_{5\times4}$ are given as follows

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \qquad [b_{ik}] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Then

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Similarly, we can also find the other products $[a_{ij}] \vee [b_{ik}]$, $[a_{ij}] \barwedge [b_{ik}]$ and $[a_{ij}] \veebar [b_{ik}]$.
Note that the commutativity is not valid for the products of soft matrices.

Proposition 8. *Let* $[a_{ij}], [b_{ik}] \in SM_{m\times n}$. *Then the following De Morgan's types of results are true.*

i. $([a_{ij}] \vee [b_{ik}])^\circ = [a_{ij}]^\circ \wedge [b_{ik}]^\circ$
ii. $([a_{ij}] \wedge [b_{ik}])^\circ = [a_{ij}]^\circ \vee [b_{ik}]^\circ$
iii. $([a_{ij}] \veebar [b_{ik}])^\circ = [a_{ij}]^\circ \barwedge [b_{ik}]^\circ$
iv. $([a_{ij}] \barwedge [b_{ik}])^\circ = [a_{ij}]^\circ \veebar [b_{ik}]^\circ$.

4. Soft max–min decision making

In this section, we construct a soft max–min decision making (*SMmDM*) method by using soft max–min decision function which is also be defined here. The method selects optimum alternatives from the set of the alternatives.

Definition 11. Let $[c_{ip}] \in SM_{m\times n^2}$, $I_k = \{p : \exists i, c_{ip} \neq 0, (k-1)n < p \leqslant kn\}$ for all $k \in I = \{1, 2, \dots, n\}$. Then soft max–min decision function, denoted *Mm*, is defined as follows

$$Mm : SM_{m\times n^2} \to SM_{m\times 1}, \qquad Mm[c_{ip}] = [\max_{k\in I}\{t_k\}]$$

where

$$t_k = \begin{cases} \min\limits_{p\in I_k}\{c_{ip}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

The one column soft matrix $Mm[c_{ip}]$ is called max–min decision soft matrix.

Definition 12. Let $U = \{u_1, u_2, \dots, u_m\}$ be an initial universe and $Mm[c_{ip}] = [d_{i1}]$. Then a subset of U can be obtained by using $[d_{i1}]$ as in the following way

$$\mathrm{opt}_{[d_{i1}]}(U) = \{u_i : u_i \in U, d_{i1} = 1\}$$

which is called an optimum set of U.

Now, by using the definitions we can construct a *SMmDM* method by the following algorithm.

Step 1: Choose feasible subsets of the set of parameters,
Step 2: construct the soft matrix for each set of parameters,
Step 3: find a convenient product of the soft matrices,
Step 4: find a max–min decision soft matrix,
Step 5: find an optimum set of U.

Note that, by the similar way, we can define soft min–max, soft min–min and soft max–max decision making methods which may be denoted by (*SmMDM*), (*SmmDM*), (*SMMDM*), respectively. One of them may be more useful than others according to the type of the problems.

5. Applications

Assume that a real estate agent has a set of different types of houses $U = \{u_1, u_2, u_3, u_4, u_5\}$ which may be characterized by a set of parameters $E = \{e_1, e_2, e_3, e_4\}$. For $j = 1, 2, 3, 4$ the parameters e_j stand for "in good location", "cheap", "modern", "large", respectively. Then we can give the following examples.

Example 6. Suppose that a married couple, Mr. X and Mrs. X, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners' parameters by using the *SMmDM* as follows.

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters.

Step 1: First, Mr. X and Mrs. X have to choose the sets of their parameters, $A = \{e_2, e_3, e_4\}$ and $B = \{e_1, e_3, e_4\}$, respectively.

Step 2: Then we can write the following soft matrices which are constructed according to their parameters.

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad [b_{ik}] = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Step 3: Now, we can find a product of the soft matrices $[a_{ij}]$ and $[b_{ik}]$ by using *And*-product as follows

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} 0&0&0&0&0&0&0&0&1&0&1&1&1&0&1&1 \\ 0&0&0&0&1&0&0&1&1&0&0&1&1&0&0&1 \\ 0&0&0&0&0&0&1&1&0&0&1&1&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1 \end{bmatrix}.$$

Here, we use And-product since both Mr. X and Mrs. X's choices have to be considered.

Step 4: We can find a max–min decision soft matrix as

$$Mm([a_{ij}] \wedge [b_{ik}]) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Step 5: Finally, we can find an optimum set of U according to $Mm([a_{ij}] \wedge [b_{ik}])$

$$\text{opt}_{Mm([a_{ij}] \wedge [b_{ik}])}(U) = \{u_1\}$$

where u_1 is an optimum house to buy for Mr. X and Mrs. X.

Note that the optimal set of U may contain more than one element.

Similarly, we can also use the other products $[a_{ij}] \vee [b_{ik}]$, $[a_{ij}] \barwedge [b_{ik}]$ and $[a_{ij}] \veebar [b_{ik}]$ for the other convenient problems.

6. Conclusion

The soft set theory has been applied to many fields from theoretical to practical. In this paper, we define soft matrices which are a matrix representation of the soft sets. We then define the set-theoretic operations of soft matrices which are more functional to improve several new results. Finally, we show that they are more practical to construct a soft decision making model on the soft set theory. We give an application for a real estate agent to choose an optimal house. It can be successfully applied to many other convenient problems that contain uncertainties.

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