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# MODELING THE TORQUE AND POWER REQUIREMENTS OF TRACTION TIRES OF HORTICULTURAL TRACTORS USING DIMENSIONAL ANALYSIS

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**Abstract-** The development of computational tools will promote the design and use of more energy-efficient tires. Two mathematical equations were developed by dimensional analysis to predict the torque and power requirements at zero net traction for traction tires (6.5-12; 7.00-18) on a hard surface. Some structural and working parameters of tire that affect the torque requirement, such as tire size, tire deflection, tire load, and rolling radius, were considered for the analysis. Experiments were conducted to study the effect of different wheel sizes and system parameters on torque and energy consumed per unit distance traveled in a soil bin**.** The ratio of tire width over tire diameter and the ratio of tire deflection over tire section height were found to be dimensionless terms radically controlling the torque and energy requirements of tires. The prediction equation closely followed the experimental results.
**Keywords**- dimensional analysis, rolling resistance of tires, tire geometry, tire deformation

## 1. INTRODUCTION

Energy efficiency has become an increasingly important consideration in world. Much effort has been expended to develop cultural systems for crops that minimize fuel usage as well as to find alternative renewable fuels for crop production.

At present, there is an attempt to better utilize the energy consumption in agricultural production, particularly for high energy requiring operations. The farm tractor consumes approximately 20 % of the total on farm energy requirements [1]. Optimizing the performance of agricultural tractors could, therefore, help to minimize energy waste. The ability to develop torque in drive axles of agricultural vehicles depends on the dynamic axle load, velocity, contact surface geometry, and the soil type and condition [2].

Research results show that from 20 to 55% of the energy delivered to the drive wheels of tractors is wasted in the traction elements. This energy is not only wasted but the resulting soil compaction created by a portion of this energy may be detrimental to crop production. This loss of energy by the pneumatic tire has prompted researchers to search for operational parameters that could improve traction efficiency [3,4].

Dimensional analysis is the method of describing a physical system by a dimensionally correct equation among certain variables. It provides advantages in reducing the number of variables which must be investigated and in formulating advantageous dimensionless variables [5,6,7].

The objective of this study was to develop mathematical models for predicting the torque and power requirement of traction tires using dimensional analysis and then to verify the model.

## 2. MODEL DEVELOPMENT

Dimensional analysis was used to develop the prediction equations for the torque and power requirements just for overcoming the rolling resistance of the self-propelled wheels of horticultural tractors. Based on the Buckingham Pi Theorem [5,6,7], the number of dimensionless and independent quantities (namely Pi terms) required to express a relationship among the variables in any physical system can be determined as follows:

$$s = n - b \tag{1}$$

where s is the number of Pi terms; n is the total number of variables; and b is the number of basic dimensions. Basic dimensions are mass (M), Length (L) and time (T).
The pertinent variables that affect tire traction performance of a self-propelled wheel with zero sinkage on a hard surface are presented in Table 1.

**Table 1**. Variable affecting the power requirement

| Symbol | Variable | Dimension | Unit |
|---|---|---|---|
| Dependent variable | | | |
| T | Torque | $ML^2T^{-2}$ | daNm |
| r | Rolling radius | L | m |
| Independent variable | | | |
| W | Normal Load | $MLT^{-2}$ | daN |
| Tire properties | | | |
| b | Tire section | L | m |
| D | Tire diameter | L | m |
| δ | Tire deflection | L | m |
| H | Tire section height | L | m |
| Constant | | | |
| v | Velocity | $LT^{-1}$ | $m \cdot s^{-1}$ |
| g | Gravity | $LT^{-2}$ | $m \cdot s^{-2}$ |

Six Pi terms are needed since there are nine variables and three basic dimensions in the system of self-propelled wheel moving on the hard surface (see Table 1). The basic dimensions of each variable are also presented in Table 1.
The torque needed to overcome the rolling resistance of the wheel can be expressed as a function of other eight variables:

$$T = f\left(b, D, \delta, h, r, W, v, g\right) \tag{2}$$

To determine Pi terms, the following equation was established:

$$\Pi_i = T^{x1} \cdot b^{x2} \cdot D^{x3} \cdot \delta^{x4} \cdot h^{x5} \cdot r^{x6} \cdot W^{x7} \cdot v^{x8} \cdot g^{x9} \tag{3}$$

where $x1 \cdots x9$ are unknowns.
Because Pi terms should not have dimension, the dimensional equation corresponding to equation 4 can be written as follows:

$$M^0L^0T^0 = \left(ML^2T^{-2}\right)^{x1} \cdot (L)^{x2} \cdot (L)^{x3} \cdot (L)^{x4} \cdot (L)^{x5} \cdot (L)^{x6} \cdot \left(MLT^{-2}\right)^{x7} . \left(LT^{-1}\right)^{x8} \cdot \left(LT^{-2}\right)^{x9} \tag{4}$$

Equation 5 were partitioned into its three components based on basic dimensions as follows:

$$\text{For M} \quad x1 + x7 = 0 \tag{5a}$$

$$\text{For L} \quad 2 \cdot x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 = 0 \tag{5b}$$

$$\text{For T} \;\; -2\cdot x1 - 2\cdot x7 - x8 - 2\cdot x9 = 0 \qquad (5c)$$

Because three equations were available for solving for the nine unknowns, three unknowns (x1,x3,x8) were kept and one of the remaining unknowns was equaled to 1 while the others were equaled to 0 to find each Pi term. The determinant of the coefficients of three variables kept should not be equal to zero to ensure that resulting Pi terms are independent [5,6]. x1, x3, and x8 obeyed this rule as shown below:

$$\begin{matrix} x1 & x3 & x8 \end{matrix}$$
$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ -2 & 0 & -1 \end{vmatrix} = -1 \qquad (6)$$

The calculation of each Pi term was given in Table 2:

**Table 2**. Calculation of Pi terms

| Assigned values of unknowns | Simplified forms of equations 5a, 5b, 5c | Pi terms |
|---|---|---|
| x2=1; x4=0; x5=0<br>x6=0; x7=0; x9=0 | x1+0=0<br>2·x1+1+x3+0+0+0+0+x8+0=0<br>-2x1-2·0-x8-2·0=0 | $\Pi_1 = \frac{b}{D}$ |
| x2=0; x4=1; x5=0<br>x6=0; x7=0; x9=0 | x1+0=0<br>2·x1+0+x3+1+0+0+0+x8+0=0<br>-2x1-2·0-x8-2·0=0 | $\Pi_2 = \frac{\delta}{D}$ |
| x2=0; x4=0; x5=1<br>x6=0; x7=0; x9=0 | x1+0=0<br>2·x1+0+x3+0+1+0+0+x8+0=0<br>-2x1-2·0-x8-2·0=0 | $\Pi_3 = \frac{h}{D}$ |
| x2=0; x4=0; x5=0<br>x6=1; x7=0; x9=0 | x1+0=0<br>2·x1+0+x3+0+0+1+0+x8+0=0<br>-2x1-2·0-x8-2·0=0 | $\Pi_4 = \frac{r}{D}$ |
| x2=0; x4=0; x5=0<br>x6=0; x7=1; x9=0 | x1+1=0<br>2·x1+0+x3+0+0+0+1+x8+0=0<br>-2x1-2·1-x8-2·0=0 | $\Pi_5 = \frac{W \cdot r}{T}$ |
| x2=0; x4=0; x5=0<br>x6=0; x7=0; x9=1 | x1+0=0<br>2·x1+0+x3+0+0+0+0+x8+1=0<br>-2x1-2·0-x8-2·1=0 | $\Pi_6 = \frac{g \cdot D}{v^2}$ |

A new set of Pi terms can be generated by changing x1, x3 and x8 partially and totally with other unknowns by guaranteeing that the determinant of their coefficients are not equal to zero. In other way, new Pi terms can be generated by multiplying and/or dividing present Pi terms with each other. In addition, a present Pi term can be reversed to make a new Pi term. But, the independency condition of Pi terms requires that any selected six Pi terms can not be generated from each other. Thus, if a new Pi term is selected for modeling, one of the present Pi terms involving in its calculation should be omitted. Some of Pi terms were transformed as shown in Table 3 to make them easy to work with.

**Table 3**. Transformation among Pi terms

| Old Pi | Transformation | New Pi |
|---|---|---|
| $\Pi_1 = \frac{b}{D}$ | No transformation | $\Pi_1 = \frac{b}{D}$ |
| $\Pi_2 = \frac{\delta}{D}$ | $\frac{\Pi_2}{\Pi_3}$ | $\Pi_2 = \frac{\delta}{h}$ |
| $\Pi_3 = \frac{h}{D}$ | $\frac{\Pi_4}{\Pi_3}$ | $\Pi_3 = \frac{r}{h}$ |
| $\Pi_4 = \frac{r}{D}$ | No transformation | $\Pi_4 = \frac{r}{D}$ |
| $\Pi_5 = \frac{W \cdot r}{T}$ | $\frac{1}{\Pi_5}$ | $\Pi_5 = \frac{T}{W \cdot r}$ |
| $\Pi_6 = \frac{g \cdot D}{v^2}$ | No transformation | $\Pi_6 = \frac{g \cdot D}{v^2}$ |

The torque is contained only in the fifth Pi terms. Thus, $\Pi_5$ is related to other five Pi terms as follows;

$$\Pi_5 = \theta(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_6) \quad (7)$$

The effect of $\Pi_6$ on $\Pi_5$ was ignored since the single velocity value was used in the tire tests and gravitational acceleration was constant. The other four Pi terms are related to the properties of tire and have similar variables. Hence, only the effects of $\Pi_1$ and $\Pi_2$ on $\Pi_5$ were considered as follows;

$$\Pi_5 = \theta(\Pi_1, \Pi_2) \quad (8)$$

In this way, the modeling process was simplified by using the shortened number of Pi terms, while all variables except forward velocity and gravitational acceleration were still included in further modeling steps. The first Pi term (aspect ratio) is about tire geometry. The second Pi term is about the tire deformation depending upon inflation pressure and the overall material strength of tires. It is known as tire deflection percentage. The fifth Pi term (power number) is about the torque and power requirements of tire. This Pi term is the only dependent Pi term considered in this study. In addition, the energy consumption of wheel per unit distance (namely power consumption rate) was calculated as follows[8]:

$$E = \frac{T}{r \cdot W} \cdot \frac{1}{1 - S} \cdot W \quad (9)$$

where E is energy consumption of tire per unit distance (watt·s·m$^{-1}$); and S is slip of tire on the ground. The slip of tires on a hard surface is practically equal to zero. The energy consumption per unit distance was used as the measure for the power requirement of tire. Its multiplication with tire forward velocity gives the exact value of the power requirement of tire.

## 3. EXPERIMENTATION

All tests were conducted in the soil bin at the Department of Agricultural Machinery, Selçuk University, Turkey by using a single-wheel agricultural tire test machine as described in the literature [9]. This machine has provisions for operating the test tires and for controlling the dynamic load.

The most common traction tires used in horticultural tractors in Turkey were selected for the study (Table 4).

**Table 4**. Specification of various tires.

| Size | Diameter (D) mm | Width (b) mm | Carcass section height (h) mm |
|---|---|---|---|
| 6.5-12 | 577 | 165 | 110 |
| 7.00-18 | 766 | 178 | 154 |

In experiments, tires were operated at dynamic loads of 300, 400, 500 and 600daN, at constant forward velocity of $0.51 m \cdot s^{-1}$. Forward velocity was measured using a speed sensor attached to test machine. The rolling radius of the tire was determined according to ASAE standard S296.2 [10]. The tire deflection percentage values (δ/h) (19, 26, 35%) that could be obtained for each load and tire within the safe inflation pressure recommended by the manufacturer were selected for study. Tires were tested on a hard surface using a single wheel tester in a soil bin for each normal load and tire deflection. The input torque was sensed by a torque transducer and was recorded using data logger.

## 4. RESULTS AND DISCUSSION

Experimental results showed a negative linear relationship between $\Pi_5$ and $\Pi_1$ as seen in Figure 1. The increase of aspect ratio ($\Pi_1$) reduced power number ($\Pi_5$). It means that torque requirement decreases with tire diameter but increases with tire width. On the other hand, there is a positive linear relationship between $\Pi_5$ and $\Pi_2$ (tire deflection) as presented in Figure 2.

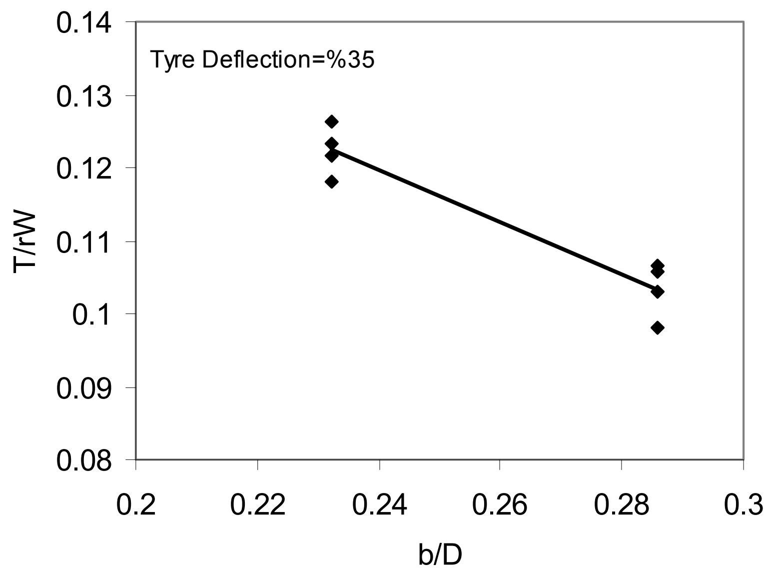


**Figure 1.** The dependence of $\Pi_5$ on $\Pi_1$

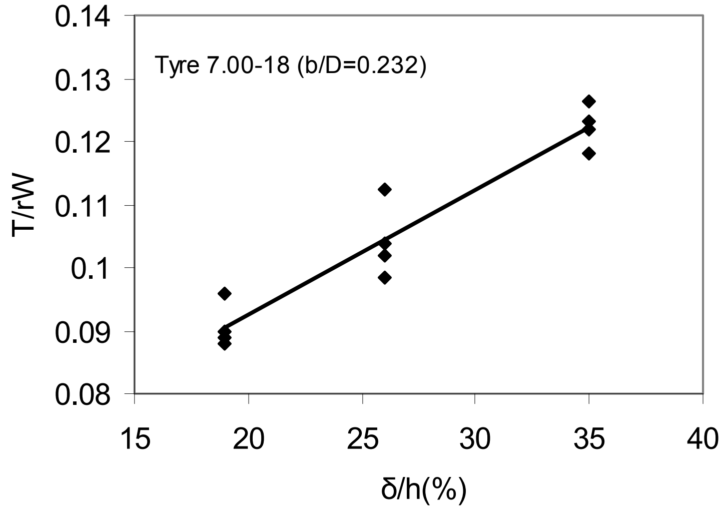


**Figure 2.** The dependence of $\Pi_5$ on $\Pi_2$

Torque requirement is expected to increase with tire deflection but to increase with tire section height. The dependence of $\Pi_5$ on $\Pi_1$ and $\Pi_2$ was mathematically described by a multivariable linear regression equation as follows:

$$\frac{T}{r \cdot W} = 0.1231 + 0.018 \cdot \frac{\delta}{h} - 0.2775 \cdot \frac{b}{D} \tag{10}$$

The statistical results about the regression equation are presented in Table 5 and Table 6. All statistical analyses were performed by using SigmaPlot® software package.

**Table 5**. Analysis of variance ($R^2$ =0.903):

| | Degrees of Freedom | Sum of Squares | Mean of Squares | F | P |
|---|---|---|---|---|---|
| Regression | 2 | 0.0045 | 0.0023 | 98.2970 | <0.0001 |
| Residual | 21 | 0.0005 | 0.0000 | | |
| Total | 23 | 0.0050 | 0.0002 | | |

**Table 6.** Results of statistical analysis on the coefficients of regression equation

| Coefficient | Value | Std. Error | t | P |
|---|---|---|---|---|
| a1 | 0.1231 | 0.0103 | 11.9044 | <0.0001 |
| a2 | 0.0018 | 0.0001 | 11.7953 | <0.0001 |
| a3 | -0.2775 | 0.0366 | -7.5806 | <0.0001 |

Analysis of variance indicated that the regression equation was statistically important (see Table 5). In addition, the coefficients of the proposed regression equation are statistically different from zero (see Table 6). Thus, power number can be successfully predicted by using aspect ratio and tire deflection percentage. The torque needed to overcome the rolling resistance of tire on the hard surface can be estimated by the following equation:

$$T = r \cdot W \cdot \left( 0.1231 + 0.018 \cdot \frac{\delta}{h} - 0.2775 \cdot \frac{b}{D} \right) \tag{11}$$

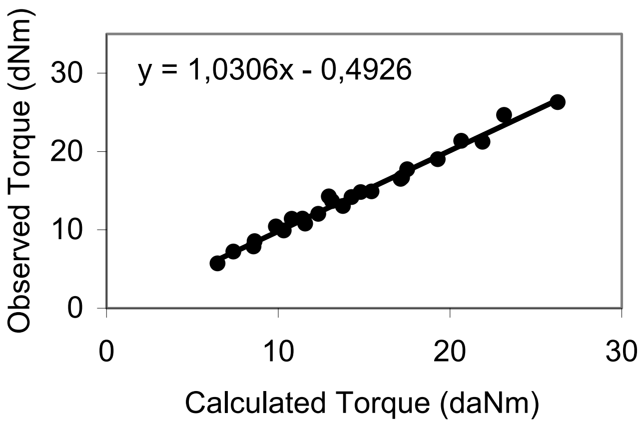


**Figure 3.** The goodness of torque prediction by equation 11.

In the case of the exact predictions of observed torque values, the slope of the line given in Figure 3 must be equal to 1. The line slope (1.03) found in this study is very close to 1. Thus, it can be said that the prediction equation is a component mathematical model to estimate the torque requirements of the traction tires of horticulture tractors.

The load carried by the tire drastically increased the power requirement of tire (Figure 4). On the other hand, the deflection of tire slightly increased the power requirement of tire.

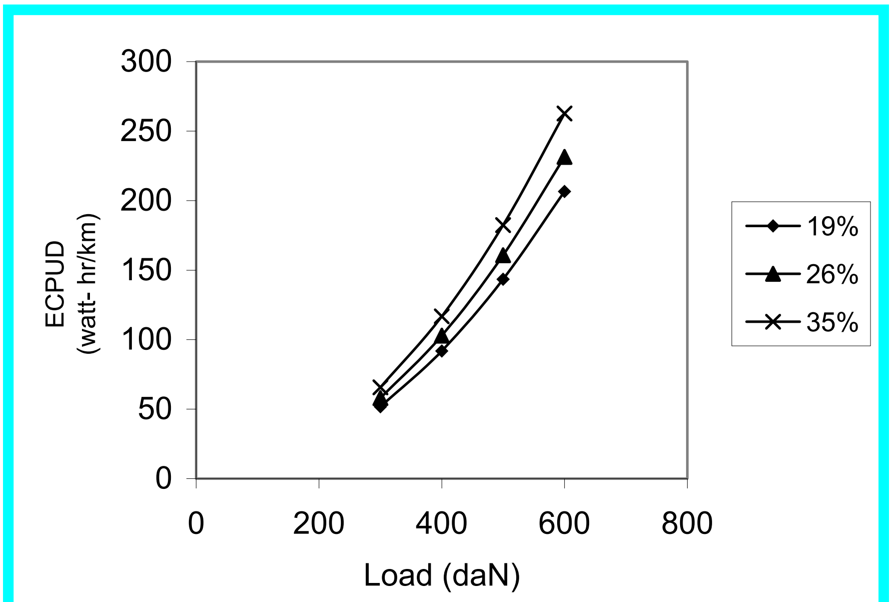


**Figure 4.** Power requirement of tire in terms of energy consumption per unit distance (ECPUD)

## 5. CONCLUSIONS

Two mathematical equations were developed to predict torque and power requirements of the traction tires of horticultural tractors by applying dimensional analysis and regression analysis over the variables describing the operational and geometric features of tires. The basic prediction equation gave results similar to the experimentally-determined torque values. To design more energy-efficient tires, the mathematical model proposed in this study can be utilized together with the other models describing the relationships between the operational and basic factors directly affecting tractors.

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